Derived equivalence classification of cluster-tilted algebras of Dynkin type E

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Abstract

We address the question of when cluster-tilted algebras of Dynkin type E are derived equivalent and as main result obtain a complete derived equivalence classification. It turns out that two cluster-tilted algebras of type E are derived equivalent if and only if their Cartan matrices represent equivalent bilinear forms over the integers which in turn happens if and only if the two algebras are connected by a sequence of "good" mutations. For type E_6 all details are given in the paper, for types E_7 and E_8 we present the results in a concise form from which our findings should easily be verifiable.

1 Introduction

Cluster algebras have been introduced by Fomin and Zelevinsky around 2000 and have enjoyed a remarkable success story in recent years. They attractively link various areas of mathematics, like combinatorics, algebraic Lie theory, representation theory, algebraic geometry and integrable systems and have applications to mathematical physics. In an attempt to 'categorify' cluster algebras (without coefficients), cluster categories have been introduced by Buan, Marsh, Reiten, Reineke, Todorov [5]. More precisely, these are orbit categories of the form $\mathcal{C}_Q = \mathrm{D}^b(KQ)/\tau^{-1}[1]$ where Q is a quiver without oriented cycles, $\mathrm{D}^b(KQ)$ is the bounded derived category of the path algebra KQ (over an algebraically closed field K) and τ and [1] are the Auslander-Reiten translation and shift functor on $\mathrm{D}^b(KQ)$, respectively. Remarkably, these cluster categories are again triangulated categories by a result of Keller [12].

Quivers of Dynkin types ADE play a special role in the theory of cluster algebras since they parametrize cluster-finite cluster algebras, by a seminal result of Fomin and Zelevinsky [9]. The corresponding cluster categories $\mathcal{C}_Q$ where Q is a Dynkin quiver are triangulated categories with finitely many indecomposable objects and their structure is well understood by work of Amiot [1].

Important objects in cluster categories are the cluster-tilting objects. A cluster-tilted algebra of type Q is by definition the endomorphism algebra of a cluster-tilting object in the cluster category $\mathcal{C}_Q$. The corresponding cluster-tilted algebras of Dynkin types A, D and E are of finite representation type and they can be constructed explicitly by quivers and relations. Namely, the quivers of the cluster-tilted algebras of Dynkin type Q are precisely the ones obtained from Q by performing finitely many quiver mutations. Moreover, in this case the quiver of a cluster-tilted algebra uniquely determines the relations [7]; we shall review the corresponding algorithm in Section 2 below.

In this paper we address the question of when two cluster-tilted algebras of Dynkin type E_6, E_7 or E_8 have equivalent derived categories. The analogous question has been settled for cluster-tilted algebras of type A_n by Buan and Vatne [8] (see also work of Murphy on the more general case of m-cluster tilted algebras of type A_n [17]) and by the first author [3] for type $\tilde{A}$. Note that the cluster-tilted algebras in these cases are gentle algebras [2]. It turns out that two cluster-tilted algebras of type A_n are derived equivalent if and only if their quivers have the same number of 3-cycles. For distinguishing such algebras up to derived equivalence one uses the determinants of the Cartan matrices; these have been determined explicitly for arbitrary gentle algebras by the second author in [11].

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A derived equivalence classification of cluster-tilted algebras of other Dynkin types D and E has been open. In this paper we settle this question for type E, i.e. we obtain a complete derived equivalence classification for cluster-tilted algebras of types E_6, E_7 and E_8.

There are two natural approaches to address derived equivalence classification problems of a given collection of algebras arising from some combinatorial data. The top-to-bottom approach is to divide these algebras into equivalence classes according to some invariants of derived equivalence, so that algebras belonging to different classes are not derived equivalent. The bottom-to-top approach is to systematically construct, based on the combinatorial data, tilting complexes yielding derived equivalences between pairs of these algebras and then to arrange these algebras into groups where any two algebras are related by a sequence of such derived equivalences. To obtain a complete derived equivalence classification one has to combine these approaches and hope that the two resulting partitions of the entire collection of algebras coincide.

The invariant of derived equivalence we use in this paper is the integer equivalence class of the bilinear form represented by the Cartan matrix of an algebra A. As this invariant is sometimes arithmetically subtle to compute directly, we instead compute the determinant of the Cartan matrix C_A and the characteristic polynomial of its asymmetry matrix $S_A = C_A C_A^{-T}$, defined whenever C_A is invertible over $\mathbb{Q}$, and encode them conveniently in a single polynomial that we call the *polynomial associated with* C_A. This quantity is generally a weaker invariant of derived equivalence, but in our case it will turn out to be enough for the classification. Note that unlike as in type A, the determinant itself is not sufficient for distinguishing the algebras up to derived equivalence.

We stress that the asymmetry matrix and its characteristic polynomial are well defined whenever the Cartan matrix is invertible over $\mathbb{Q}$, even without having any categorical meaning, as follows from [16, Section 3.3]. In the special case when A has finite global dimension, the asymmetry matrix S_A, or better minus its transpose $-C_A^{-1} C_A^T$, is related to the Coxeter transformation which does carry categorical meaning, and its characteristic polynomial is known as the Coxeter polynomial of the algebra.

The tilting complexes we use are inspired by quiver mutations in the following sense. For a vertex, we consider all the incoming arrows and build, based on this combinatorial data, a two-term complex of projective modules, see also similar constructions in [15], [19]. We call a mutation at a vertex "good" if the corresponding complex is a tilting complex and moreover its endomorphism algebra is the cluster-tilted algebra of the mutated quiver. In other words, a "good" mutation produces a derived equivalence between the corresponding cluster-tilted algebras. Of course, there are also "bad" mutations, for two reasons: the complex might not be a tilting complex or even if it is, its endomorphism algebra might not be a cluster-tilted algebra.

It turns out that for cluster-tilted algebras of type E the two approaches can be successfully combined to give a complete derived equivalence classification. More precisely, our main result is the following.

Theorem 1.1. *The following conditions are equivalent for two cluster-tilted algebras A and A' of Dynkin type E:*

(a) A and A' have the same associated polynomial;

(b) The Cartan matrices of A and A' represent equivalent bilinear forms over $\mathbb{Z}$;

(c) A and A' are derived equivalent;

(d) A and A' are connected via a sequence of "good" mutations.

In addition to the above general statement we make the derived equivalence classification explicit by providing complete lists of the algebras contained in each derived equivalence class (up to sink/source equivalence).

Note that the implication (c) $\Rightarrow$ (b) holds in general for any two (finite-dimensional) algebras A and A', and that the implication (b) $\Rightarrow$ (a) holds whenever the associated polynomials are defined, i.e. when the Cartan matrices are invertible over $\mathbb{Q}$. Moreover, for cluster-tilted algebras the implication (d) $\Rightarrow$ (c) is evident from the definition.

We also note that since the cluster-tilted algebras we consider involve only zero- and commutativity-relations, they can in fact be defined over any commutative ring K. The combinatorial nature of our construction of tilting complexes via "good" mutations will then imply that such algebras corresponding

to quivers in the same class will be derived equivalent for any K. Therefore one may view the derived equivalences arising from "good" mutations as "universal", being independent on the auxiliary algebraic data (specified by K).

Let us briefly describe the above main result in some more detail. For precise definitions of the cluster-tilted algebras involved we refer to Sections 3 (type E_6), A/B (type E_7), and C/D (type E_8) below. In the following tables we list the associated polynomials of the cluster-tilted algebras, and also the total number of algebras in each derived equivalence class.

For type E_6 the mutation class consists of 67 quivers. The corresponding cluster-tilted algebras turn out to fall into six derived equivalence classes as follows.

Derived equivalence classes for type E_6			
Associated polynomial	#	Associated polynomial	#
$x^6 - x^5 + x^3 - x + 1$	20	$3(x^6 + x^3 + 1)$	19
$2(x^6 - x^4 + 2x^3 - x^2 + 1)$	16	$4(x^6 + x^4 + x^2 + 1)$	7
$2(x^6 - 2x^4 + 4x^3 - 2x^2 + 1)$	3	$4(x^6 + x^5 - x^4 + 2x^3 - x^2 + x + 1)$	2

For type E_7 the mutation class consists of 416 quivers. The derived equivalence classes of the corresponding cluster-tilted algebras are again characterized by the associated polynomials; there are 14 classes in total, given as follows.

Derived equivalence classes for type E_7			
Associated polynomial	#	Associated polynomial	#
$x^7 - x^6 + x^4 - x^3 + x - 1$	64	$4(x^7 + x^6 - 2x^5 + 2x^4 - 2x^3 + 2x^2 - x - 1)$	2
$2(x^7 - x^5 + 2x^4 - 2x^3 + x^2 - 1)$	32	$4(x^7 + x^5 - x^4 + x^3 - x^2 - 1)$	56
$2(x^7 - x^5 + x^4 - x^3 + x^2 - 1)$	72	$4(x^7 + x^5 - 2x^4 + 2x^3 - x^2 - 1)$	8
$2(x^7 - 2x^5 + 4x^4 - 4x^3 + 2x^2 - 1)$	8	$5(x^7 + x^5 - x^4 + x^3 - x^2 - 1)$	17
$3(x^7 - 1)$	124	$6(x^7 + x^6 - x^4 + x^3 - x - 1)$	11
$4(x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x - 1)$	16	$6(x^7 + x^5 - x^2 - 1)$	1
$4(x^7 + x^6 - x^5 - x^4 + x^3 + x^2 - x - 1)$	4	$8(x^7 + x^6 + x^5 - x^4 + x^3 - x^2 - x - 1)$	1

For type E_8 the mutation class consists of 1574 quivers. The corresponding cluster-tilted algebras turn out to fall into 15 different derived equivalence classes which are characterized as follows.

Derived equivalence classes for type E_8			
Associated polynomial	#	Associated polynomial	#
$x^8 - x^7 + x^5 - x^4 + x^3 - x + 1$	128	$4(x^8 + x^6 - x^5 + 2x^4 - x^3 + x^2 + 1)$	221
$2(x^8 - x^6 + 2x^5 - 2x^4 + 2x^3 - x^2 + 1)$	64	$4(x^8 + x^6 - 2x^5 + 4x^4 - 2x^3 + x^2 + 1)$	22
$2(x^8 - x^6 + x^5 + x^3 - x^2 + 1)$	256	$5(x^8 + x^6 + x^4 + x^2 + 1)$	167
$2(x^8 - 2x^6 + 4x^5 - 4x^4 + 4x^3 - 2x^2 + 1)$	16	$6(x^8 + x^6 + x^5 + x^3 + x^2 + 1)$	38
$3(x^8 + x^4 + 1)$	384	$6(x^8 + x^7 + 2x^4 + x + 1)$	118
$4(x^8 + x^7 - x^6 + x^5 + x^3 - x^2 + x + 1)$	72	$8(x^8 + 2x^7 + 2x^4 + 2x + 1)$	4
$4(x^8 + x^7 - x^6 + 2x^4 - x^2 + x + 1)$	48	$8(x^8 + x^7 + x^6 + 2x^4 + x^2 + x + 1)$	24
$4(x^8 + x^7 - 2x^6 + 2x^5 + 2x^3 - 2x^2 + x + 1)$	12		

The paper is organized as follows. In Section 2 we collect some background material; in particular we recall the notion of quiver mutation, describe the results of Buan, Marsh and Reiten on cluster-tilted algebras of finite representation type, review the fundamental results on derived equivalences and then discuss invariants of derived equivalence such as the equivalence class of the Euler form, in particular leading to the determinant of the Cartan matrix and the characteristic polynomial of its asymmetry matrix as derived invariants.

In Section 3 we discuss derived equivalences for cluster-tilted algebras of Dynkin type E_6 in detail. The quivers of these algebras are given by those in the mutation class of type E_6; this mutation class can easily be reproduced by the reader using Keller's software [13]. We first give in Section 3 a list of

the derived equivalence classes, sorted by the associated polynomial which is the crucial invariant for our purposes. We also give the Cartan matrix of one representative in each class which we shall need later in our computations.

As main result of this section we prove the main Theorem 1.1 for type E_6. To this end we have to find explicit tilting complexes for the cluster-tilted algebras of type E_6 and we have to determine their endomorphism rings. The necessary calculations are carried out in detail in Sections 3.2 - 3.4. The tilting complexes we use are closely related to quiver mutations at a single vertex, and we explain this construction in Section 3.1.

For types E_7 and E_8 we have followed a different strategy of presentation since the number of algebras involved becomes very large. We first list the algebras but without drawing the quivers; again, the quivers can be found using Keller's software. We then present the results on derived equivalences for cluster-tilted algebras of types E_7 and E_8 in a very concise form which is explained at the beginning of the respective sections. For each group of algebras with the same associated polynomial we then provide tilting complexes and list their endomorphism rings, but without giving any details on the calculations. However, we hope that we have provided enough information so that interested readers should easily be able to check our findings.

Acknowledgement

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2 Preliminaries

2.1 Quiver mutations

A *quiver* is a finite directed graph Q, consisting of a finite set of vertices Q_0 and a finite set of arrows Q_1 between them. A fundamental concept in the theory of Fomin and Zelevinsky's cluster algebras is mutation; for quivers this takes the following shape.

Definition 2.1. Let Q be a quiver without loops and oriented 2-cycles. For vertices i, j, let a_{ij} denote the number of arrows from i to j, where $a_{ij} < 0$ means that there are $-a_{ij}$ arrows from j to i.

The *mutation of Q at the vertex k* yields a new quiver $\mu_k(Q)$ obtained from Q by the following procedure:

1. Add a new vertex k^*.
2. For all vertices $i \neq j$, different from k, such that $a_{ij} \geq 0$, set the number of arrows a'_{ij} from i to j in $\mu_k(Q)$ as follows:
   if $a_{ik} \geq 0$ and $a_{kj} \geq 0$, then $a'_{ij} := a_{ij} + a_{ik}a_{kj}$;
   if $a_{ik} \leq 0$ and $a_{kj} \leq 0$, then $a'_{ij} := a_{ij} - a_{ik}a_{kj}$.
3. For any vertex i, replace all arrows from i to k with arrows from k^* to i, and replace all arrows from k to i with arrows from i to k^*.
4. Remove the vertex k.

Two quivers are called *mutation equivalent* if one can be obtained from the other by a finite sequence of mutations. The *mutation class* of a quiver Q is the class of all quivers mutation equivalent to Q. It is known from the seminal results of Fomin and Zelevinsky [9] that the mutation class of a Dynkin quiver Q is finite.

2.2 Cluster-tilted algebras of finite representation type

Cluster-tilted algebras arise as endomorphism algebras of cluster-tilting objects in a cluster category, see [6]. For the special case of Dynkin quivers the cluster-tilted algebras are known to be of finite representation type. Moreover, by a result of Buan, Marsh and Reiten [7] they can be described as quivers with relations by a simple combinatorial recipe to be recalled below. As a consequence, a cluster-tilted algebra of Dynkin type is uniquely determined by its quiver.

Let Q be a quiver and throughout this paper let K be an algebraically closed field. We can form the *path algebra* KQ, where the basis of KQ is given by all paths in Q, including trivial paths e_i of length zero at each vertex i of Q. Multiplication in KQ is defined by concatenation of paths. Our convention is to compose paths from right to left. For any path α in Q let $s(\alpha)$ denote its start vertex and $t(\alpha)$ its end vertex. Then the product of two paths α and β is defined to be the concatenated path $\alpha\beta$ if $s(\alpha) = t(\beta)$. The unit element of KQ is the sum of all trivial paths, i.e., $1_{KQ} = \sum\limits_{i \in Q_0} e_i$.

We recall some background from [7]. An oriented cycle in a quiver is called *full* if it does not contain any repeated vertices and if the subquiver generated by the cycle contains no other arrows. If there is an arrow $i \to j$ in a quiver Q then a path from j to i is called *shortest path* if the induced subquiver is a full cycle.

We now describe cluster-tilted algebras of Dynkin type by a quiver with relations, i.e. in the form KQ/I where Q is a finite quiver and I is some admissible ideal in the path algebra KQ. Recall that the quivers associated with cluster-tilted algebras of Dynkin type are precisely the quivers in the mutation class of the corresponding Dynkin quiver.

Relations are linear combinations $k_1\omega_1 + \cdots + k_m\omega_m$ of paths ω_i in Q, all starting in the same vertex and ending in the same vertex, and with each k_i non-zero in K. If $m = 1$, we call the relation a *zero-relation.* If $m = 2$ and $k_1 = 1$, $k_2 = -1$, and we call it a *commutativity-relation* (and say that the paths ω_1 and ω_2 commute). It will turn out that for cluster-tilted algebras of Dynkin type the ideal I can be generated by only using zero-relations and commutativity-relations. Finally, a relation ρ is called *minimal* if whenever $\rho = \sum_i \beta_i \circ \rho_i \circ \gamma_i$, where ρ_i is a relation for every i, then there is an index j such that both β_j and γ_j are scalars.

Proposition 2.2 (Buan, Marsh and Reiten [7])**.** *A cluster-tilted algebra A of finite representation type is of the form $A = KQ/I$, where Q is mutation equivalent to a Dynkin quiver and where the ideal I can be described as follows. Let i and j be vertices in Q.*

1. *The ideal I is generated by minimal zero-relations and minimal commutativity-relations.*

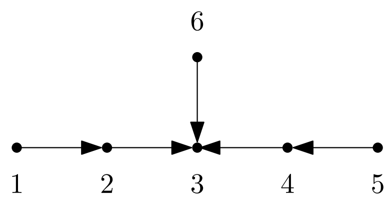
2. *Assume there is an arrow $i \to j$. Then there are at most two shortest paths from j to i.*

   i) If there is exactly one, then this is a minimal zero-relation.

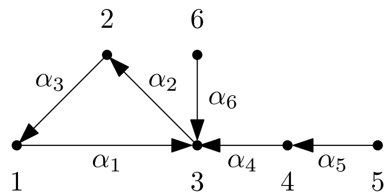
   ii) If there are two, ω and μ, then ω and μ are not zero in A and there is a minimal relation $\omega - \mu$.

3. *Up to multiplication by non-zero elements of K there are no other minimal zero-relations or commutativity-relations than the ones coming from 2.*

Example 2.3. We consider the following quiver Q of type E_6

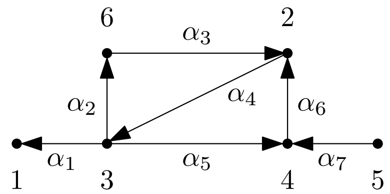


If we mutate at vertex 2, we get the following quiver $Q' := \mu_2(Q)$



The corresponding cluster-tilted algebra is of the form $A = KQ'/I$ where I is generated by the zero-relations $\alpha_1\alpha_3$, $\alpha_2\alpha_1$ and $\alpha_3\alpha_2$ (and there are no commutativity-relations).

Mutating the latter quiver at the vertex 3 leads to the quiver $Q'' := \mu_3(Q')$



Here, the ideal of relations of the corresponding cluster-tilted algebra is generated by the zero-relations $\alpha_2\alpha_4$, $\alpha_5\alpha_4$, $\alpha_4\alpha_3$ and $\alpha_4\alpha_6$ and the commutativity-relation $\alpha_3\alpha_2 = \alpha_6\alpha_5$.

2.3 Tilting complexes and derived equivalences

In this section we briefly review the fundamental results on derived equivalences. All algebras are assumed to be finite-dimensional K-algebras.

For a K-algebra A the bounded derived category of A-modules is denoted by $\mathrm{D}^b(A)$. Recall that two algebras A, B are called derived equivalent if $\mathrm{D}^b(A)$ and $\mathrm{D}^b(B)$ are equivalent as triangulated categories. By a famous theorem of Rickard [18] derived equivalences can be found using the concept of tilting complexes.

Definition 2.4. A *tilting complex* T over A is a bounded complex of finitely generated projective A-modules satisfying the following conditions:

i) $\mathrm{Hom}_{\mathrm{D}^b(A)}(T, T[i]) = 0$ for all $i \neq 0$, where [1] denotes the shift functor in $\mathrm{D}^b(A)$;

ii) the category $\mathrm{add}(T)$ (i.e. the full subcategory consisting of direct summands of direct sums of T) generates the homotopy category $\mathrm{K}^b(P_A)$ of projective A-modules as a triangulated category.

We can now formulate Rickard's seminal result.

Theorem 2.5 (Rickard [18])**.** *Two algebras A and B are derived equivalent if and only if there exists a tilting complex T for A such that the endomorphism algebra $\mathrm{End}_{\mathrm{D}^b(A)}(T) \cong B$.*

2.4 The equivalence class of the Euler form as derived invariant

Let A be a finite-dimensional algebra over a field K and let $P_1, \ldots, P_n$ be a complete collection of non-isomorphic indecomposable projective A-modules (finite-dimensional over K). The *Cartan matrix* of A is then the $n \times n$ matrix C_A defined by $(C_A)_{ij} = \dim_K \mathrm{Hom}(P_j, P_i)$.

Denote by $\mathrm{per}\, A$ the triangulated category of *perfect* complexes of A-modules inside the derived category of A, that is, complexes (quasi-isomorphic) to finite complexes of finitely generated projective A-modules. The Grothendieck group $K_0(\mathrm{per}\, A)$ is a free abelian group on the generators $[P_1], \ldots, [P_n]$, and the expression

$$\langle X, Y \rangle = \sum_{r \in \mathbb{Z}} (-1)^r \dim_K \mathrm{Hom}_{\mathrm{per}\, A}(X, Y[r])$$

is well defined for any $X, Y \in \mathrm{per}\, A$ and induces a bilinear form on $K_0(\mathrm{per}\, A)$, known as the *Euler form*, whose matrix with respect to the basis of projectives is C_A^T.

The following proposition is well known. For the convenience of the reader, we give the short proof, see also the *proof* of Proposition 1.5 in [4].

Proposition 2.6. *Let A and B be two finite-dimensional, derived equivalent algebras. Let n denote by number of their non-isomorphic indecomposable projectives. Then the matrices C_A and C_B represent equivalent bilinear forms over $\mathbb{Z}$, that is, there exists $P \in \mathrm{GL}_n(\mathbb{Z})$ such that $PC_AP^T = C_B$.*

Proof. Indeed, by [18], if A and B are derived equivalent, then $\operatorname{per} A$ and $\operatorname{per} B$ are equivalent as triangulated categories. Now any triangulated functor $F : \operatorname{per} A \to \operatorname{per} B$ induces a linear map from $K_0(\operatorname{per} A)$ to $K_0(\operatorname{per} B)$. When F is also an equivalence, this map is an isomorphism of the Grothendieck groups preserving the Euler forms. Thus, if $[F]$ denotes the matrix of this map with respect to the bases of indecomposable projectives, then $[F]^T C_B [F] = C_A$. □

In general, to decide whether two integral bilinear forms are equivalent is a very subtle arithmetical problem. Therefore, it is useful to introduce somewhat weaker invariants that are computationally easier to handle. In order to do this, assume further that C_A is invertible over $\mathbb{Q}$. In this case one can consider the rational matrix $S_A = C_A C_A^{-T}$ (here C_A^{-T} denotes the inverse of the transpose of C_A), known in the theory of non-symmetric bilinear forms as the *asymmetry* of C_A.

Proposition 2.7. *Let A and B be two finite-dimensional, derived equivalent algebras with invertible (over $\mathbb{Q}$) Cartan matrices. Then we have the following assertions, each implied by the preceding one:*

1. *There exists $P \in \mathrm{GL}_n(\mathbb{Z})$ such that $PC_AP^T = C_B$.*
2. *There exists $P \in \mathrm{GL}_n(\mathbb{Z})$ such that $PS_AP^{-1} = S_B$.*
3. *There exists $P \in \mathrm{GL}_n(\mathbb{Q})$ such that $PS_AP^{-1} = S_B$.*
4. *The matrices S_A and S_B have the same characteristic polynomial.*

For proofs and discussion, see for example [16, Section 3.3]. Since the determinant of an integral bilinear form is invariant under equivalence, we can combine it with the characteristic polynomial $p_{S_A}(x)$ of the asymmetry matrix S_A to obtain a discrete invariant of derived equivalence, namely $(\det C_A) \cdot p_{S_A}(x)$. We call this invariant the *polynomial associated with C_A*.

Remark 2.8. The matrix $S_A = C_A C_A^{-T}$ (or better, minus its transpose $-C_A^{-1} C_A^T$) is related to the *Coxeter transformation* which has been widely studied in the case when A has finite global dimension (so that C_A is invertible over $\mathbb{Z}$). It is the K-theoretic shadow of the Serre functor and the related Auslander-Reiten translation in the derived category. The characteristic polynomial is then known as the *Coxeter polynomial* of the algebra.

Remark 2.9. In general, S_A might have non-integral entries. However, when the algebra A is *Gorenstein*, the matrix S_A is integral, which is an incarnation of the fact that the injective modules have finite projective resolutions. By a result of Keller and Reiten [14], this is the case for the cluster-tilted algebras in question.

2.5 Computations of Cartan matrices

Let $A = KQ/I$ be an algebra given by a quiver $Q = (Q_0, Q_1)$ with relations. Since $\sum_{i \in Q_0} e_i$ is the unit element in A we get a decomposition $A = A \cdot 1 = \bigoplus_{i \in Q_0} Ae_i$, hence the (left) A-modules $P_i := Ae_i$ are the indecomposable projective A-modules, and the Cartan matrix $C_A = (c_{ij})$ of A is the n-by-n matrix whose entries are $c_{ij} = \dim_K \operatorname{Hom}_A(P_j, P_i)$, where $n = |Q_0|$. Any homomorphism $\varphi : Ae_j \to Ae_i$ of left A-modules is uniquely determined by $\varphi(e_j) \in e_j A e_i$, the K-vector space generated by all paths in Q from vertex i to vertex j that are non-zero in A. In particular, we have $c_{ij} = \dim_K e_j A e_i$, i.e., computing entries of the Cartan matrix for A reduces to counting paths in Q.

For cluster-tilted algebras of Dynkin type the entries of the Cartan matrix can only be 0 or 1, as the following result shows.

Proposition 2.10 (Buan, Marsh, Reiten [7]). *Let A be a cluster-tilted algebra of finite representation type. Then $\dim_K \operatorname{Hom}_A(P_j, P_i) \leq 1$ for any two indecomposable projective A-modules P_i and P_j.*

Example 2.11. We have a look at the quivers in Example 2.3 again, and compute the Cartan matrices of the corresponding cluster-tilted algebras.

For the Dynkin quiver Q of type E_6 with the above orientation we get the following Cartan matrix $\begin{pmatrix} 1&1&1&0&0&0\\ 0&1&1&0&0&0\\ 0&0&1&0&0&0\\ 0&0&1&1&0&0\\ 0&0&1&1&1&0\\ 0&0&1&0&0&1 \end{pmatrix}$ since there are no zero- or commutativity-relations.

For the quiver Q' obtained by mutation from Q at vertex 2, the corresponding Cartan matrix C' has the form $C' = \begin{pmatrix} 1&0&1&0&0&0\\ 1&1&0&0&0&0\\ 0&1&1&0&0&0\\ 0&1&1&1&0&0\\ 0&1&1&1&1&0\\ 0&1&1&0&0&1 \end{pmatrix}$ for KQ/I since the paths from vertex 1 to 2, from 2 to 3 and from 3 to 1 are zero.

Finally, for the quiver Q'' obtained from Q' by mutating at vertex 3, the cluster-tilted algebra has Cartan matrix $C'' = \begin{pmatrix} 1&0&0&0&0&0\\ 1&1&1&0&0&0\\ 1&1&1&1&0&1\\ 0&1&0&1&0&0\\ 0&1&0&1&1&0\\ 0&1&0&0&0&1 \end{pmatrix}$. Note that the two paths from vertex 3 to vertex 2 (over 4 or 6) are the same since we have the commutativity-relation $\alpha_3\alpha_2 = \alpha_6\alpha_5$.

For calculating the endomorphism ring $\operatorname{End}_{D^b(A)}(T)$ of a tilting complex T over the algebra A, we can use the following statement which explicitly gives the Cartan matrix of the endomorphism ring in terms of the tilting complex and the Cartan matrix of A.

Proposition 2.12. *Let T be a tilting complex over A with endomorphism algebra $B = \operatorname{End}_{D^b(A)}(T)$, and let $T_1, \dots, T_n$ be the indecomposable direct summands of T. Then the Cartan matrix C_B of B is given by $C_B = PC_AP^T$, where $P = (p_{ij})_{i,j=1}^n$ is the matrix defined by*

$$[T_i] = \sum_{j=1}^{n} p_{ij}[P_j]$$

(that is, its i-th row is the class of the summand T_i in $K_0(\operatorname{per} A)$ written in the basis $[P_1], \dots, [P_n]$).

Example 2.13. Continuing Example 2.11, let $T = T_1 \oplus \cdots \oplus T_6$ be the complex over the cluster-tilted algebra corresponding to Q' defined by

$$T_i = \begin{cases} P_i & \text{if } i \neq 3 \\ P_3 \to P_1 \oplus P_4 \oplus P_6 & \text{if } i = 3, \end{cases}$$

where the P_i are in degree 0 for $i \neq 3$ and P_3 is in degree -1.

Then T is a tilting complex and the corresponding matrix P is given by

$$P = \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 1&0&-1&1&0&1\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix},$$

so that $C'' = PC'P^T$. In fact, $\operatorname{End} T$ is isomorphic to the cluster-tilted algebra corresponding to Q'', see Section 3.2.1.

It is sometimes convenient to use the following alternating sum formula, arising from the fact that for a bounded complex $X = (X^r)$ of projective modules, we have $[X] = \sum (-1)^r [X^r]$ in $K_0(\operatorname{per} A)$.

Proposition 2.14 (Happel [10]). *For an algebra A let $X = (X^r)_{r\in\mathbb{Z}}$ and $Y = (Y^s)_{s\in\mathbb{Z}}$ be bounded complexes of projective A-modules. Then*

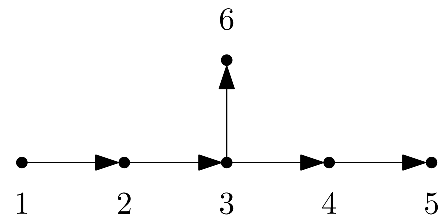
$$\sum_i (-1)^i \dim \operatorname{Hom}_{D^b(A)}(X, Y[i]) = \sum_{r,s} (-1)^{r-s} \dim \operatorname{Hom}_A(X^r, Y^s).$$

In particular, if X and Y are direct summands of the same tilting complex, then

$$\dim \operatorname{Hom}_{D^b(A)}(X,Y) = \sum_{r,s} (-1)^{r-s} \dim \operatorname{Hom}_A(X^r, Y^s).$$

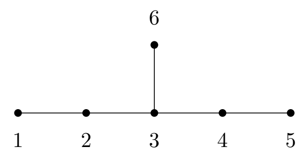
3 Derived equivalences of cluster-tilted algebras of type E_6

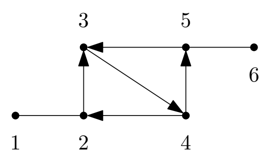
For the mutation class of E_6 we start with the following quiver

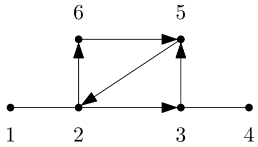
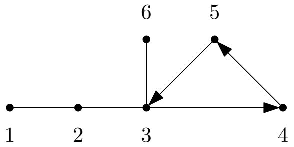
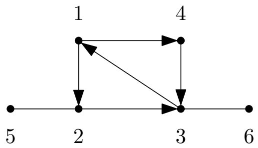


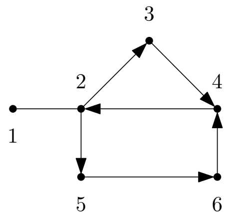
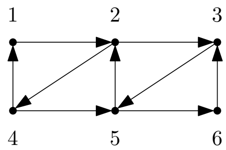
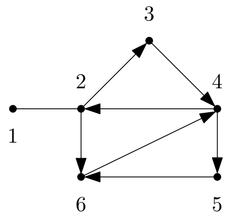
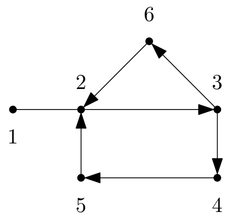
and determine all quivers which can be obtained from it by a finite number of mutations. For this, one can use the software of B. Keller [13]. The mutation class of E_6 consists of 67 quivers. For the purpose of derived equivalence classifications of the corresponding cluster-tilted algebras it suffices to consider the quivers up to sink/source equivalence, and there are 21 quivers up to sink/source equivalence. We can divide the corresponding cluster-tilted algebras into six groups by computing the polynomials associated with their Cartan matrices. Recall from the introduction that these associated polynomials are obtained by multiplying the determinant of the Cartan matrix by the characteristic polynomial of its asymmetry matrix. It will turn out that these six groups form the six derived equivalence classes of the cluster-tilted algebras of type E_6.

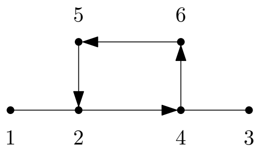
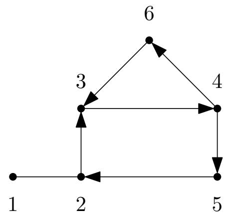
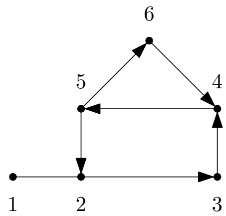
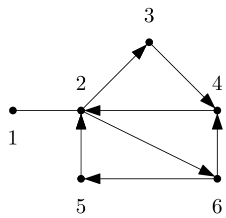
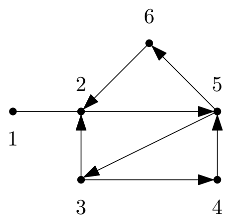
We list in the table below all quivers in the mutation class of type E_6. For each group of sink/source equivalent quivers we give only one picture where certain arrows are replaced by undirected lines; this has to be read that these lines can take any orientation. We also give the Cartan matrix of the corresponding cluster-tilted algebra of one particular representative in each group of sink/source equivalent quivers. From this Cartan matrices one can easily read off to which orientation of the undirected lines it corresponds; in fact, for a line between vertices i and j the arrow is going from i to j if the (i,j) entry in the Cartan matrix is non-zero, and from j to i otherwise. We sort the 21 classes of cluster-tilted algebras of type E_6 (up to sink/source equivalence) according to their associated polynomials, and number them according to the output of B. Keller's software [13], i.e. the cluster-tilted algebras are denoted by A_{number}.

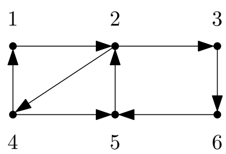
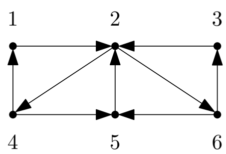
$\boldsymbol{x^6 - x^5 + x^3 - x + 1}$		
no.	quiver Q	Cartan matrix
1		$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

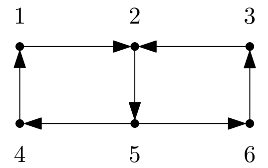
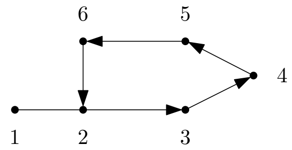
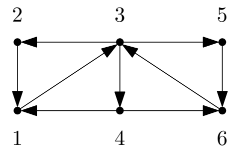
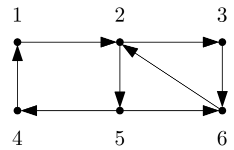
$\boldsymbol{2(x^6 - 2x^4 + 4x^3 - 2x^2 + 1)}$		
no.	quiver Q	Cartan matrix
11		$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

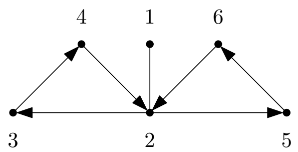
$\mathbf{2(x^6 - x^4 + 2x^3 - x^2 + 1)}$		
no.	quiver Q	Cartan matrix
2		$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
7		$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$
12		$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

$\mathbf{3(x^6 + x^3 + 1)}$		
no.	quiver Q	Cartan matrix
3		$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$
4		$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$
6		$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$
10		$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$

no.	quiver Q	Cartan matrix
14		$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$
16		$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$
19		$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$
20		$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$
21		$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$

$\mathbf{4(x^6+x^4+x^2+1)}$		
no.	quiver Q	Cartan matrix
5		$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$
9		$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$

no.	quiver Q	Cartan matrix
13		$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$
15		$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$
17		$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$
18		$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$

$\mathbf{4(x^6+x^5-x^4+2x^3-x^2+x+1)}$		
no.	quiver Q	Cartan matrix
8		$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$

The rest of this section is devoted to proving Theorem 1.1 for type E_6. To this end we shall explicitly construct suitable tilting complexes and determine their endomorphism algebras. Note that the class of cluster-tilted algebras is not closed under derived equivalences, so one carefully has to choose suitable tilting complexes in order to get another cluster-tilted algebra as endomorphism algebra.

3.1 From vertices to complexes

Since we deal with left modules and read paths from right to left, a non-zero path from vertex i to j gives a homomorphism $P_j \to P_i$ by right multiplication. Thus, two arrows $\alpha : i \to j$ and $\beta : j \to k$ give a path $\beta\alpha$ from i to k and a homomorphism $\alpha\beta : P_k \to P_i$.

Let A be a cluster-tilted algebra corresponding to a quiver Q, and let k be a vertex of Q. Consider all the arrows $j \to k$ ending at k, and define a complex $T^{(k)}$ of projective A-modules by $T^{(k)} = \bigoplus_i T_i^{(k)}$ (the sum runs over all the vertices i), with

$$T_i^{(k)} = \begin{cases} P_i & \text{if } i \neq k \\ P_k \to \bigoplus_{j \to k} P_j & \text{if } i = k \end{cases}$$

where the P_i are in degree 0 for $i \neq k$, while P_k is in degree -1.

We call the mutation at the vertex k *good* if $T^{(k)}$ is a tilting complex and moreover, $\mathrm{End}_{D^b(A)}(T^{(k)})$ is the cluster-tilted algebra corresponding to the quiver $\mu_k(Q)$ obtained from Q by mutating at the vertex

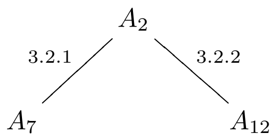
k. Thus, a good mutation yields a derived equivalence between the corresponding cluster-tilted algebras. Note that “bad” mutations can occur for two reasons: the complex $T^{(k)}$ might not be a tilting complex or even if it is, its endomorphism algebra might not be a cluster-tilted algebra.

Since $P_k[1]$ is isomorphic in the homotopy category $\mathrm{K}^b(P_A)$ to the cone of $\bigoplus_{j\to k} T_j^{(k)} \to T_i^{(k)}$, we see that all the indecomposable projectives lie in the triangulated subcategory generated by the summands of $T^{(k)}$. Thus, condition ii) of Definition 2.4 of a tilting complex is satisfied for the complex $T^{(k)}$. For checking condition i) of Definition 2.4 it is sufficient to prove that $\mathrm{Hom}_{\mathrm{D}^b(A)}(T^{(k)}, T^{(k)}[1]) = 0$ and $\mathrm{Hom}_{\mathrm{D}^b(A)}(T^{(k)}, T^{(k)}[-1]) = 0$ since the complex $T^{(k)}$ is concentrated in only two consecutive degrees.

3.2 Derived equivalences for polynomial $2(x^6 - x^4 + 2x^3 - x^2 + 1)$

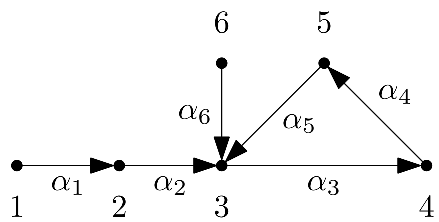
In this section we prove that all cluster-tilted algebras of type E_6 with associated polynomial as given in the title are indeed connected by a sequence of good mutations, and hence in particular are derived equivalent. The proof is divided into several subsections where in each subsection for two cluster-tilted algebras a suitable tilting complex for a good mutation is constructed, and the endomorphism algebra is determined.

For the convenience of the reader we provide the following figure which displays what is proven in each subsection and from which it should be convenient to check that indeed all cluster-tilted algebras with the relevant associated polynomials are covered.



3.2.1 A_7 is derived equivalent to A_2

Let A_7 be the cluster-tilted algebra corresponding to the quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex of projective A_7-modules corresponding to the vertex 3, as defined in Section 3.1. Explicitly, $T_i : 0 \to P_i \to 0$ for $i \in \{1, 2, 4, 5, 6\}$ are complexes concentrated in degree zero and $T_3 : 0 \to P_3 \overset{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} P_2 \oplus P_5 \oplus P_6 \to 0$ in degrees -1 and 0.

Now we want to show that T is a tilting complex and we begin with possible maps $T_3 \to T_3[1]$ and $T_3 \to T_3[-1]$,

$$\begin{array}{ccccccccc}
 & & 0 & \to & P_3 & \overset{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} & P_2 \oplus P_5 \oplus P_6 & \to & 0 \\
 & & & & \downarrow \psi & & & & \\
0 & \to & P_3 & \overset{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} & P_2 \oplus P_5 \oplus P_6 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_3 & \overset{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} & P_2 \oplus P_5 \oplus P_6 & \to & 0
\end{array}$$

where $\psi \in \mathrm{Hom}(P_3, P_2 \oplus P_5 \oplus P_6)$ and $(\alpha_2, 0, 0)$, $(0, \alpha_5, 0)$, $(0, 0, \alpha_6)$ is a basis of this three-dimensional space of homomorphisms. The homomorphism ψ is homotopic to zero and in the second case there is no non-zero homomorphism $P_2 \oplus P_5 \oplus P_6 \to P_3$.

Now consider possible maps $T_3 \to T_j[-1]$, $j \neq 3$. These maps are given by a map of complexes as follows

$$\begin{array}{ccccccc} 0 & \to & P_3 & \stackrel{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} & P_2 \oplus P_5 \oplus P_6 & \to & 0 \\ & & \downarrow & & & & \\ 0 & \to & Q & \to & 0 & & \end{array}$$

where Q could be either P_1, P_2, P_5, P_6 or direct sums of these. Note that there is no non-zero homomorphism $P_3 \to P_4$ since this is a zero-relation in the quiver of A_7. There exist non-zero homomorphisms of complexes, but they are all homotopic to zero since every homomorphism from P_3 to P_1, P_2, P_5 or P_6 starts with α_2, α_5 or α_6, up to scalars. Thus, every homomorphism $P_3 \to Q$ can be factored through the map $(\alpha_2, \alpha_5, \alpha_6) : P_3 \to P_2 \oplus P_5 \oplus P_6$. Directly from the definition we see that $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 2, 4, 5, 6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

Finally, we have to consider maps $T_j \to T_3[1]$ for $j \neq 3$. But these are given as follows

$$\begin{array}{ccccccc} 0 & \to & P_4 & \to & 0 & & \\ & & \downarrow \alpha_3 & & & & \\ 0 & \to & P_3 & \stackrel{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} & P_2 \oplus P_5 \oplus P_6 & \to & 0 \end{array}$$

since $\mathrm{Hom}(P_j, P_3) = 0$ for $j = 1, 2, 5$ and $j = 6$. But the concatenation of $(\alpha_2, \alpha_5, \alpha_6)$ and α_3 is not zero since $\alpha_2\alpha_3 \neq 0$ and $\alpha_6\alpha_3 \neq 0$. So the only homomorphism of complexes $T_j \to T_3[1]$, $j \neq 3$, is the zero map.

It follows that T is a tilting complex for A_7, and by Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(A_7)}(T)$ is derived equivalent to A_7. We want to show that E is isomorphic to one possible orientation of the class of algebras A_2. Using the alternating sum formula of Proposition 2.14 we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$.

Now we define homomorphisms of complexes between the summands of T which correspond to the arrows of the following quiver

$$\begin{tikzcd} & 6 \arrow[rr, "\alpha_6\alpha_3"] & & 4 \arrow[dl, "\delta"] & \\ 5 & 3 \arrow[l, "\beta"'] \arrow[u, "\gamma"] \arrow[rr, "\alpha"'] & & 2 \arrow[u, "\alpha_2\alpha_3"'] & 1 \arrow[l, "\alpha_1"] \end{tikzcd}$$

Note that this is a quiver from the class denoted A_2 (up to renumbering of vertices). First we have the embeddings $\alpha := (\mathrm{id}, 0, 0) : T_2 \to T_3$, $\beta := (0, \mathrm{id}, 0) : T_5 \to T_3$ and $\gamma := (0, 0, \mathrm{id}) : T_6 \to T_3$ (in degree zero). Then we define $\delta : T_3 \to T_4$ by the map $(0, \alpha_4, 0) : P_2 \oplus P_5 \oplus P_6 \to P_4$ in degree 0. This is a homomorphism of complexes since $\alpha_4\alpha_5 = 0$ in A_7. Moreover, we have the homomorphisms $\alpha_2\alpha_3 : T_4 \to T_2$ and $\alpha_6\alpha_3 : T_4 \to T_6$. Finally, we also have the homomorphism α_1 as before. Note that the homomorphisms correspond to the reversed arrows.

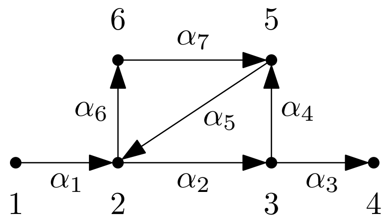
Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $(0, \alpha_6\alpha_3\alpha_4, 0)$ and $(0, \alpha_2\alpha_3\alpha_4, 0)$ are zero since they were zero in A_7. As we can see, the paths from vertex 4 to vertex 2 and to vertex 6 are zero. There is one commutativity-relation between vertex 3 and vertex 4 left. This is given by the two homomorphisms from T_4 to the first and third summand of T_3. These are indeed the same paths since $(0, 0, \alpha_6\alpha_3)$ is homotopic to $(\alpha_2\alpha_3, 0, 0)$.

$$\begin{tikzcd} & 0 \arrow[r] & P_4 \arrow[dl, "\alpha_3"'] \arrow[d, "\alpha_2\alpha_3"'] \arrow[d, shift left=3, "\alpha_6\alpha_3"] \arrow[r] & 0 \\ 0 \arrow[r] & P_3 \arrow[r, "{(\alpha_2, \alpha_5, \alpha_6)}"'] & P_2 \oplus P_5 \oplus P_6 \arrow[r] & 0 \end{tikzcd}$$

From this we can conclude that $E \cong A_2$ and thus, A_7 and A_2 are derived equivalent.

3.2.2 A_2 is derived equivalent to A_{12}

Now we consider a cluster-tilted algebra A_2 with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex from Section 3.1 corresponding to the vertex 2. Explicitly, the complexes $T_i : 0 \to P_i \to 0$ for $i \in \{1,3,4,5,6\}$ are concentrated in degree zero and the complex $T_2 : 0 \to P_2 \stackrel{(\alpha_1,\alpha_5)}{\longrightarrow} P_1 \oplus P_5 \to 0$ in degrees -1 and 0.

To show that T is a tilting complex we begin with possible maps $T_2 \to T_2[1]$ and $T_2 \to T_2[-1]$

$$\begin{array}{ccccccccc} & & 0 & \to & P_2 & \stackrel{(\alpha_1,\alpha_5)}{\longrightarrow} & P_1 \oplus P_5 & \to & 0 \\ & & & & \downarrow \psi & & & & \\ 0 & \to & P_2 & \stackrel{(\alpha_1,\alpha_5)}{\longrightarrow} & P_1 \oplus P_5 & \to & 0 & & \\ & & & & \downarrow \varphi & & & & \\ & & 0 & \to & P_2 & \stackrel{(\alpha_1,\alpha_5)}{\longrightarrow} & P_1 \oplus P_5 & \to & 0 \end{array}$$

Here $\psi \in \mathrm{Hom}(P_2, P_1 \oplus P_5)$ and $(\alpha_1, 0), (0, \alpha_5)$ is a basis of this two-dimensional space. But then ψ is homotopic to zero (as we can easily see). In the second case $(0, \alpha_6\alpha_7) = (0, \alpha_2\alpha_4)$ is a basis of the space of homomorphisms between $P_1 \oplus P_5$ and P_2. Hence, φ is not a homomorphism of complexes since $\alpha_1\alpha_6\alpha_7 = \alpha_1\alpha_2\alpha_4 \neq 0$.

Now consider possible maps $T_2 \to T_j[-1]$, $j \neq 2$. These are given by maps of complexes as follows

$$\begin{array}{ccccccc} 0 & \to & P_2 & \stackrel{(\alpha_1,\alpha_5)}{\longrightarrow} & P_1 \oplus P_5 & \to & 0 \\ & & \downarrow & & & & \\ 0 & \to & Q & \to & 0 & & \end{array}$$

where Q could be either P_1, P_5 or direct sums of these. Note that there are no non-zero homomorphisms $P_2 \to P_3$, $P_2 \to P_4$ and $P_2 \to P_6$ since these are zero-relations in the quiver of A_2. There exist non-zero homomorphisms of complexes, but they are all homotopic to zero since every homomorphism from P_2 to P_1 or P_5 starts with a scalar multiple of α_1 or α_5. Thus, every homomorphism $P_2 \to Q$ can be factored through the map $(\alpha_1, \alpha_5) : P_2 \to P_1 \oplus P_5$. Hence, $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1,3,4,5,6\}$ and thus $\mathrm{Hom}(T, T[-1]) = 0$.

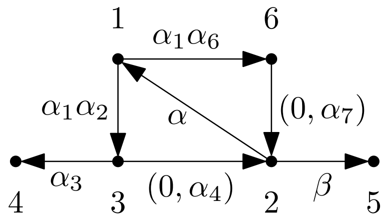
Finally, we have to consider maps $T_j \to T_2[1]$ for $j \neq 2$. These are given as follows

$$\begin{array}{ccccccc} 0 & \to & Q & \to & 0 & & \\ & & \downarrow & & & & \\ 0 & \to & P_2 & \stackrel{(\alpha_1,\alpha_5)}{\longrightarrow} & P_1 \oplus P_5 & \to & 0 \end{array}$$

where Q can be either P_3, P_4, P_5, P_6 or direct sums of these since $\mathrm{Hom}(P_1, P_2) = 0$. But no non-zero map can be zero when composed with both α_1 and α_5 since the paths $\alpha_2\alpha_1$ and $\alpha_6\alpha_1$ are not zero. So the only homomorphism of complexes $T_j \to T_2[1]$, $j \neq 2$, is the zero map.

It follows that T is a tilting complex for A_2, and by Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(A_2)}(T)$ is derived equivalent to A_2. We show that E is isomorphic to A_{12}. Using the alternating sum formula of Proposition 2.14 we compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$ which coincides with the Cartan matrix of A_{12} (up to permutation).

Now we have to define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver in the class A_{12} of the form



First we have the embeddings $\alpha := (\mathrm{id}, 0) : T_1 \to T_2$ and $\beta := (0, \mathrm{id}) : T_5 \to T_2$ (in degree zero). Moreover, we have the homomorphisms $\alpha_1\alpha_2 : T_3 \to T_1,\ \alpha_1\alpha_6 : T_6 \to T_1,\ (0, \alpha_4) : T_2 \to T_3$ and $(0, \alpha_7) : T_2 \to T_6$. Finally, we also have the homomorphism α_3 as before. Note that the homomorphisms correspond to the reversed arrows.

Now we have to show that these homomorphisms satisfy the defining relations of A_{12}, up to homotopy. Clearly, the concatenation of $(0, \alpha_4)$ and α and the concatenation of $(0, \alpha_7)$ and α are zero-relations. It is easy to see, that the two paths from vertex 1 to vertex 2 are the same since $\alpha_1\alpha_6\alpha_7 = \alpha_1\alpha_2\alpha_4$. The two paths from vertex 2 to vertex 3 and from vertex 2 to vertex 6 are zero since $(\alpha_1\alpha_2, 0)$ and $(\alpha_1\alpha_6, 0)$ are homotopic to zero. Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_{12}. From this we can conclude that $E \cong A_{12}$ and thus, A_2 and A_{12} are derived equivalent.

Hence, we get derived equivalences between A_2, A_7 and A_{12}.

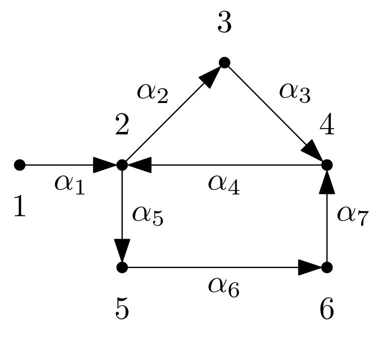
3.3 Derived equivalences for polynomial $3(x^6 + x^3 + 1)$

We again provide the following figure which displays what is proven in each subsection.

$$\begin{tikzcd}
A_{16} \arrow[r, dash, "3.3.5"] & A_6 \arrow[r, dash, "3.3.4"] & A_3 \arrow[r, dash, "3.3.1"] \arrow[d, dash, "3.3.2"'] & A_{20} \arrow[r, dash, "3.3.1"] \arrow[d, dash, "3.3.3"] & A_{10} \arrow[r, dash, "3.3.4"] & A_{21} \arrow[r, dash, "3.3.5"] & A_{19} \\
 & & A_4 & A_{14} & & &
\end{tikzcd}$$

3.3.1 A_3 and A_{10} are derived equivalent to A_{20}

Now we consider the cluster-tilted algebra A_3 with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex corresponding to the vertex 5. Namely, $T_i : 0 \to P_i \to 0$ for $i \in \{1, 2, 3, 4, 6\}$ are complexes concentrated in degree zero and $T_5 : 0 \to P_5 \xrightarrow{\alpha_5} P_2 \to 0$ is a complex concentrated in degrees -1 and 0.

Now we want to show that T is a tilting complex. We begin with possible maps $T_5 \to T_5[1]$ and $T_5 \to T_5[-1]$,

$$\begin{array}{ccccccccc} & & 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \\ & & & & \downarrow \alpha_5 & & & & \\ 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 & & \\ & & & & \downarrow 0 & & & & \\ & & 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \end{array}$$

where α_5 is a basis of the space of homomorphisms between P_5 and P_2. The homomorphism α_5 is homotopic to zero and in the second case there is no non-zero homomorphism $P_2 \to P_5$ (as we can see in the Cartan matrix of A_3).

Now consider possible maps $T_5 \to T_j[-1],\ j \neq 5$. These are given by maps of complexes as follows

$$\begin{array}{ccccccc} 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \\ & & \downarrow & & & & \\ 0 & \to & Q & \to & 0 & & \end{array}$$

where Q could be either P_1, P_2, P_4 or direct sums of these. Note that there are no non-zero homomorphisms $P_5 \to P_3$ and $P_5 \to P_6$ since these are zero-relations in the quiver of A_3. There exist non-zero homomorphisms of complexes. But they are all homotopic to zero since every homomorphism from P_5 to P_1, P_2 or P_4 starts with a scalar multiple of α_5. Thus, every homomorphism $P_5 \to Q$ can be factored through the map $\alpha_5 : P_5 \to P_2$. Directly from the definition we see that $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 2, 3, 4, 6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

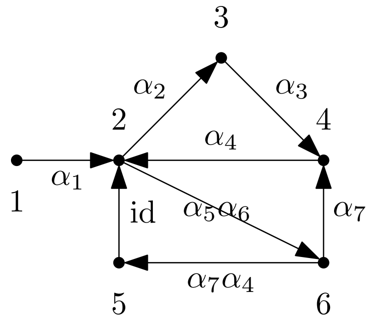
Finally, we have to consider maps $T_j \to T_5[1]$ for $j \neq 5$. These are given as follows

$$\begin{array}{ccccccc} 0 & \to & Q & \to & 0 & & \\ & & \downarrow & & & & \\ 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \end{array}$$

where Q can be either P_4, P_6 or direct sums of these since $\mathrm{Hom}(P_j, P_5) = 0$ for $j = 1, 2$ and $j = 3$. But no non-zero map can be zero when composed with α_5 since the path $\alpha_7\alpha_6\alpha_5 = \alpha_3\alpha_2 \neq 0$. So the only homomorphism of complexes $T_j \to T_5[1],\ j \neq 5$, is the zero map.

It follows that T is a tilting complex for A_3, and by Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(A_3)}(T)$ is derived equivalent to A_3. We show that E is isomorphic to A_{20}. Using the alternating sum formula of Proposition 2.14 we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$.

Now we have to define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver in the class of A_{20} and show that these homomorphisms satisfy the defining relations of A_{20}, up to homotopy.



First we have the embedding $\mathrm{id} : T_2 \to T_5$ (in degree zero). Moreover, we have the homomorphisms $\alpha_5\alpha_6 : T_6 \to T_2$ and $\alpha_7\alpha_4 : T_5 \to T_6$. Finally, we also have homomorphisms $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_7 as before.

Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $\alpha_3\alpha_4,\ \alpha_4\alpha_2,\ \alpha_4\alpha_5\alpha_6$ and $\alpha_5\alpha_6\alpha_7\alpha_4$ are zero since they were zero in A_3. As we can see, the two paths from vertex 6 to vertex 2 are the same, i.e., we here have the right commutativity-relation. There is also another commutativity-relation $\alpha_2\alpha_3 = \alpha_5\alpha_6\alpha_7$ between vertex 2 and 4 since these are the same paths in A_3. The concatenation of id and $\alpha_5\alpha_6$ yields to a zero-relation since the homomorphism $\alpha_5\alpha_6$ is homotopic to zero.

Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_{20}. We have shown that they satisfy the defining relations of A_{20} and that the Cartan matrices of E and A_{20} coincide. From this we can conclude that $E \cong A_{20}$ and thus, A_3 and A_{20} are derived equivalent. Since A_{20} is sink/source equivalent to its opposite algebra, A_{20} is also derived equivalent to $A_3^{\mathrm{op}} = A_{10}$. Hence, we get derived equivalences between A_3, A_{10} and A_{20}.

3.3.2 A_3 is derived equivalent to A_4

The second complex for A_3 is the one corresponding to the vertex 2. Namely, $T = \bigoplus_{i=1}^6 T_i$ with $T_i : 0 \to P_i \to 0$ for $i \in \{1,3,4,5,6\}$ (in degree zero) and $T_2 : 0 \to P_2 \xrightarrow{(\alpha_1,\alpha_4)} P_1 \oplus P_4 \to 0$ in degrees -1 and 0.

For showing that T is a tilting complex, we begin with possible maps $T_2 \to T_2[1]$ and $T_2 \to T_2[-1]$,

$$\begin{array}{ccccccccc}
 & & 0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_4)} & P_1 \oplus P_4 & \to & 0 \\
 & & & & \downarrow \psi & & & & \\
0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_4)} & P_1 \oplus P_4 & \to & 0 & & \\
 & & & & \downarrow \varphi & & & & \\
 & & 0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_4)} & P_1 \oplus P_4 & \to & 0
\end{array}$$

Here $\psi \in \mathrm{Hom}(P_2, P_1 \oplus P_4)$ and $(\alpha_1, 0),\ (0, \alpha_4)$ is a basis of this two-dimensional space. But then ψ is homotopic to zero (as we can easily see). In the second case $(0, \alpha_2\alpha_3) = (0, \alpha_5\alpha_6\alpha_7)$ is a basis of the space of homomorphisms between $P_1 \oplus P_4$ and P_2. Hence, φ is not a homomorphism of complexes since $\alpha_1\alpha_2\alpha_3 = \alpha_1\alpha_5\alpha_6\alpha_7 \neq 0$.

Now consider possible maps $T_2 \to T_j[-1],\ j \neq 2$. These are given by maps of complexes as follows

$$\begin{array}{cccccc}
0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_4)} & P_1 \oplus P_4 & \to 0 \\
 & & \downarrow & & & \\
0 & \to & Q & \to & 0 &
\end{array}$$

where Q could be either P_1, P_4, P_6 or direct sums of these. Note that there are no non-zero homomorphisms $P_2 \to P_3$ and $P_2 \to P_5$ since these are zero-relations in the quiver of A_3. There exist non-zero homomorphisms of complexes, but they are all homotopic to zero since every homomorphism from P_2 to P_1, P_4 or P_6 starts with a scalar multiple of α_1 or α_4. Thus, every homomorphism $P_2 \to Q$ can be factored through the map $(\alpha_1, \alpha_4) : P_2 \to P_1 \oplus P_4$. Hence, $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1,3,4,5,6\}$ and thus $\mathrm{Hom}(T, T[-1]) = 0$.

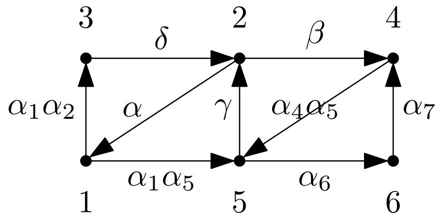
Finally, we have to consider maps $T_j \to T_2[1]$ for $j \neq 2$. These are given as follows

$$\begin{array}{cccccc}
0 & \to & Q & \to & 0 & \\
 & & \downarrow & & & \\
0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_4)} & P_1 \oplus P_4 & \to 0
\end{array}$$

where Q can be either P_3, P_4, P_5, P_6 or direct sums of these since $\mathrm{Hom}(P_1, P_2) = 0$. But no non-zero map can be zero when composed with both α_1 and α_4 since the paths $\alpha_2\alpha_1$ and $\alpha_5\alpha_1$ are not zero. So the only homomorphism of complexes $T_j \to T_2[1],\ j \neq 2$, is the zero map.

It follows that T is a tilting complex for A_3, and by Rickard's theorem, $E := \mathrm{End}_{D^b(A_3)}(T)$ is derived equivalent to A_3. We want to show that E is isomorphic to A_4 and use the alternating sum formula of Proposition 2.14 for computing the Cartan matrix of E. This Cartan matrix is given as follows $\begin{pmatrix} 1&1&1&1&1&1 \\ 1&1&0&1&1&0 \\ 0&1&1&1&0&0 \\ 0&0&0&1&1&0 \\ 0&1&0&1&1&1 \\ 0&0&0&1&0&1 \end{pmatrix}$ and it is the Cartan matrix of A_4 up to permutation.

Now we have to define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver in the class of A_4 of the form



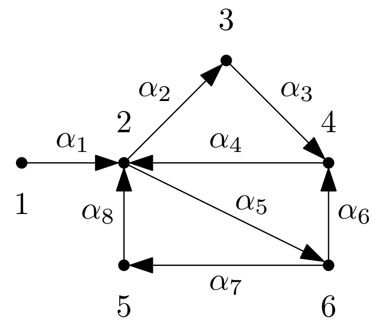
First $\alpha := (\mathrm{id}, 0) : T_1 \to T_2$ and $\beta := (0, \mathrm{id}) : T_4 \to T_2$ are the embeddings, $\gamma : T_2 \to T_5$ is defined by the map $(0, \alpha_6\alpha_7) : P_1 \oplus P_4 \to P_5$ and $\delta : T_2 \to T_3$ is defined by $(0, \alpha_3) : P_1 \oplus P_4 \to P_3$ (in degree 0). These are a homomorphisms of complexes since $\alpha_6\alpha_7\alpha_4 = 0$ and $\alpha_3\alpha_4 = 0$ in A_3. Moreover, we have the homomorphisms $\alpha_1\alpha_2 : T_3 \to T_1$, $\alpha_1\alpha_5 : T_5 \to T_1$ and $\alpha_4\alpha_5 : T_5 \to T_4$. Finally, we also have homomorphisms α_6 and α_7 as before.

Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $\alpha_4\alpha_5\alpha_6$, $\alpha_7\alpha_4\alpha_5$ and $(0, \alpha_4\alpha_5\alpha_6\alpha_7)$ are zero since they were zero in A_3. As we can see, the two paths from vertex 5 to vertex 4 are the same, i.e., we here have the right commutativity-relation. There are also two other commutativity-relations left. First $(0, \alpha_1\alpha_2\alpha_3) = (0, \alpha_1\alpha_5\alpha_6\alpha_7)$ between vertex 1 and 2 is one of them since these are the same paths in A_3. Secondly, the two paths from vertex 2 to vertex 5 are the same since $(\alpha_1\alpha_5, 0)$ is homotopic to $(0, \alpha_4\alpha_5)$. It is easy to see that the concatenation of γ and α and the concatenation of δ and α are zero-relations. Finally, the path from vertex 2 to vertex 3 is zero since $(\alpha_1\alpha_2, 0)$ is homotopic to zero.

Thus, we can conclude that $E \cong A_4$ and thus, A_3 and A_4 are derived equivalent. Since $A_4 = A_4^{\mathrm{op}}$, A_4 is also derived equivalent to $A_3^{\mathrm{op}} = A_{10}$. Hence, we get derived equivalences between A_3, A_4, A_{10} and A_{20}.

3.3.3 A_{20} is derived equivalent to A_{14}

Consider A_{20} with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex corresponding to the vertex 4. Explicitly, $T_i : 0 \to P_i \to 0$ for $i \in \{1, 2, 3, 5, 6\}$ are concentrated in degree zero and $T_4 : 0 \to P_4 \overset{(\alpha_3,\alpha_6)}{\longrightarrow} P_3 \oplus P_6 \to 0$ is concentrated in degrees -1 and 0.

To show that T is a tilting complex we begin with possible maps $T_4 \to T_4[1]$ and $T_4 \to T_4[-1]$:

$$\begin{array}{ccccccccc}
 & & 0 & \to & P_4 & \overset{(\alpha_3,\alpha_6)}{\longrightarrow} & P_3 \oplus P_6 & \to & 0 \\
 & & & & \downarrow \psi & & & & \\
0 & \to & P_4 & \overset{(\alpha_3,\alpha_6)}{\longrightarrow} & P_3 \oplus P_6 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_4 & \overset{(\alpha_3,\alpha_6)}{\longrightarrow} & P_3 \oplus P_6 & \to & 0
\end{array}$$

where $\psi \in \mathrm{Hom}(P_4, P_3 \oplus P_6)$ and $(\alpha_3, 0)$, $(0, \alpha_6)$ is a basis of this two-dimensional space. The first homomorphism is homotopic to zero (as we can easily see). In the second case there is no non-zero homomorphism $P_3 \oplus P_6 \to P_4$ (as we can see in the Cartan matrix of A_{20}).

Now we consider possible maps $T_4 \to T_j[-1]$ and $T_j \to T_4[1]$, $j \neq 4$. These are given by maps of complexes as follows

$$\begin{array}{ccccccc} 0 & \to & P_4 & \overset{(\alpha_3,\alpha_6)}{\longrightarrow} & P_3 \oplus P_6 & \to & 0 \\ & & \downarrow & & & & \\ 0 & \to & Q & \to & 0 & & \end{array}$$

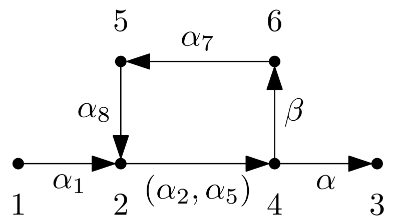
where Q could be either P_1, P_2, P_3, P_6 or direct sums of these and

$$\begin{array}{ccccccc} 0 & \to & R & \to & 0 & & \\ & & \downarrow & & & & \\ 0 & \to & P_4 & \overset{(\alpha_3,\alpha_6)}{\longrightarrow} & P_3 \oplus P_6 & \to & 0 \end{array}$$

where R can be P_2 since $\mathrm{Hom}(P_j, P_4) = 0$ for $j = 1, 3, 5$ and $j = 6$. In the first case, there exist non-zero homomorphisms of complexes, but they are all homotopic to zero since every homomorphism from P_4 to P_1, P_2, P_3 or P_6 starts with a scalar multiple of α_3 or α_6. Thus, every homomorphism $P_4 \to Q$ can be factored through the map $(\alpha_3, \alpha_6) : P_4 \to P_3 \oplus P_6$. In the second case, the only homomorphism of complexes $T_2 \to T_4[1]$ is the zero map since $\alpha_6\alpha_4 \neq 0$.

It follows that T is a tilting complex for A_{20}, and by Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(A_{20})}(T)$ is derived equivalent to A_{20}. We claim that E is isomorphic to A_{14}. Using the alternating sum formula of Proposition 2.14 we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$.

Now we define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver of A_{14}.



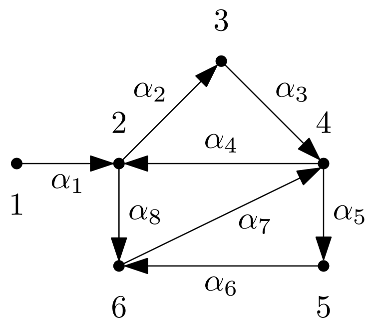
First we have the embeddings $\alpha := (\mathrm{id}, 0) : T_3 \to T_4$ and $\beta := (0, \mathrm{id}) : T_6 \to T_4$ (in degree zero). Moreover, we have the homomorphism $(\alpha_2, \alpha_5) : T_4 \to T_2$. Finally, we also have the homomorphisms α_1, α_7 and α_8 as before.

Now we have to show that these homomorphisms satisfy the defining relations of A_{14}, up to homotopy. Clearly, the homomorphisms $(\alpha_7\alpha_8\alpha_2, \alpha_7\alpha_8\alpha_5)$, $(0, \alpha_8\alpha_5)$ and $(0, \alpha_5\alpha_7)$ in the 4$-$cycle are zero since they were zero in A_{20}. The concatenation of β, α_7 and α_8 yields to a zero-relation since the homomorphism $(0, \alpha_7\alpha_8)$ is homotopic to zero.

Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_{14}. From this we can conclude that $E \cong A_{14}$ and thus, A_{20} and A_{14} are derived equivalent. Hence, we get derived equivalences between A_3, A_4, A_{10}, A_{14} and A_{20}.

3.3.4 A_6 is derived equivalent to A_3

Consider A_6 with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex corresponding to the vertex 5. That is, $T_i : 0 \to P_i \to 0$ for $i \in \{1,2,3,4,6\}$ are complexes concentrated in degree zero and $T_5 : 0 \to P_5 \xrightarrow{\alpha_5} P_4 \to 0$ in degrees -1 and 0.

For showing that T is a tilting complex we begin with possible maps $T_5 \to T_5[1]$ and $T_5 \to T_5[-1]$,

$$
\begin{array}{ccccccccc}
 & & 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_4 & \to & 0 \\
 & & & & \downarrow \alpha_5 & & & & \\
0 & \to & P_5 & \xrightarrow{\alpha_5} & P_4 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_4 & \to & 0
\end{array}
$$

Here α_5 is a basis of the space of homomorphisms between P_5 and P_4. Then α_5 is homotopic to zero (as we can easily see). In the second case there is no non-zero homomorphism $P_4 \to P_5$.

Now consider possible maps $T_5 \to T_j[-1],\ j \neq 5$. These are given by maps of complexes as follows

$$
\begin{array}{ccccccc}
0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \\
 & & \downarrow & & & & \\
0 & \to & Q & \to & 0 & &
\end{array}
$$

where Q could be either P_3, P_4 or direct sums of these. Note that there are no non-zero homomorphisms $P_5 \to P_1$, $P_5 \to P_2$ and $P_5 \to P_6$ since these are zero-relations in the quiver of A_6. There exist non-zero homomorphisms of complexes between P_5 and P_3 or P_4, but they are all homotopic to zero since every homomorphism starts with a scalar multiple of α_5. Thus, every homomorphism $P_5 \to Q$ can be factored through the map $\alpha_5 : P_5 \to P_4$. We see that $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1,2,3,4,6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

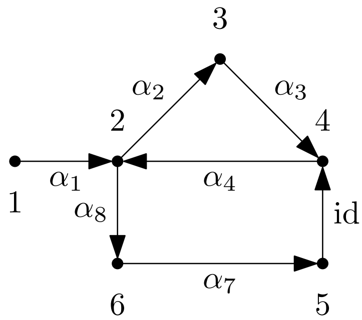
Finally, we have to consider maps $T_j \to T_5[1]$ for $j \neq 5$. These are given as follows

$$
\begin{array}{ccccccc}
0 & \to & P_6 & \to & 0 & & \\
 & & \downarrow \alpha_6 & & & & \\
0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0
\end{array}
$$

since $\mathrm{Hom}(P_j, P_5) = 0$ for $j = 1,2,3$ and $j = 4$. But the composition $\alpha_5\alpha_6 \neq 0$. So the only homomorphism of complexes $T_j \to T_5[1],\ j \neq 5$, is the zero map.

It follows that T is a tilting complex for A_6, and by Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(A_6)}(T)$ is derived equivalent to A_6. We want to show that E is isomorphic to A_3. Using the alternating sum formula of Proposition 2.14 we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$ which coincides with the Cartan matrix of A_3 (up to permutation).

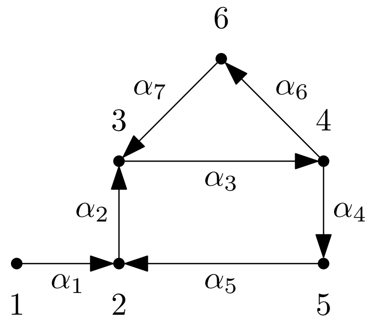
Now we have to define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver of A_3.



First we have the embedding id : $T_4 \to T_5$ (in degree zero). Moreover, we have the homomorphisms $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_7$ and α_8 as before. Since all the relations are the same as in A_6 we have shown that they satisfy the defining relations of A_3. From this we can conclude that $E \cong A_3$ and thus, A_3 and A_6 are derived equivalent. Since A_6^{op} is sink/source equivalent to A_{21}, A_{21} is also derived equivalent to $A_3^{\mathrm{op}} = A_{10}$. Hence, we get derived equivalences between A_3, A_4, A_6, A_{10}, A_{14}, A_{20} and A_{21}.

3.3.5 A_{16} is derived equivalent to A_6

Consider A_{16} with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex corresponding to the vertex 5, namely $T_i : 0 \to P_i \to 0$ for $i \in \{1, 2, 3, 4, 6\}$ are concentrated in degree zero and $T_5 : 0 \to P_5 \xrightarrow{\alpha_4} P_4 \to 0$ is concentrated in degrees -1 and 0.

Now we want to show that T is a tilting complex and we begin with possible maps $T_5 \to T_5[1]$ and $T_5 \to T_5[-1]$,

$$\begin{array}{ccccccccc} & & 0 & \to & P_5 & \xrightarrow{\alpha_4} & P_4 & \to & 0 \\ & & & & \downarrow \alpha_4 & & & & \\ 0 & \to & P_5 & \xrightarrow{\alpha_4} & P_4 & \to & 0 & & \\ & & & & \downarrow 0 & & & & \\ & & 0 & \to & P_5 & \xrightarrow{\alpha_4} & P_4 & \to & 0 \end{array}$$

Here α_4 is a basis of the space of homomorphisms between P_5 and P_4. The homomorphism α_4 is homotopic to zero and in the second case there is no non-zero homomorphism $P_4 \to P_5$ (as we can see in the Cartan matrix of A_{16}).

Now consider possible maps $T_5 \to T_j[-1]$, $j \neq 5$. These are given by maps of complexes as follows

$$\begin{array}{ccccccc} 0 & \to & P_5 & \xrightarrow{\alpha_4} & P_4 & \to & 0 \\ & & \downarrow & & & & \\ 0 & \to & Q & \to & 0 & & \end{array}$$

where Q could be either P_3, P_4 or direct sums of these. Note that there are no non-zero homomorphisms $P_5 \to P_1$, $P_5 \to P_2$ and $P_5 \to P_6$ since these are zero-relations in the quiver of A_{16}. There exist non-zero homomorphisms of complexes, but they are all homotopic to zero since every homomorphism from P_5 to P_3 or P_4 starts with a scalar multiple of α_4. Thus, every homomorphism $P_5 \to Q$ can be factored

through the map $\alpha_4 : P_5 \to P_4$. Hence, $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 2, 3, 4, 6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

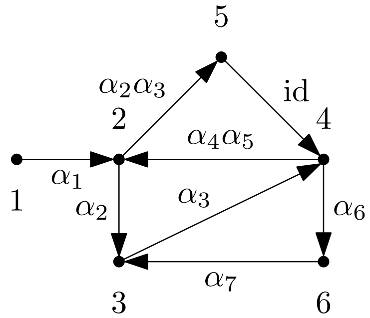
Finally, we have to consider maps $T_j \to T_5[1]$ for $j \neq 5$. These are given as follows

$$\begin{array}{ccccccc} 0 & \to & Q & \to & 0 & & \\ & & \downarrow & & & & \\ 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \end{array}$$

where Q can be either P_2, P_3 or direct sums of these since $\mathrm{Hom}(P_j, P_5) = 0$ for $j = 1, 4$ and $j = 6$. But no non-zero map can be zero when composed with α_4 since the path $\alpha_2\alpha_5\alpha_4 = \alpha_7\alpha_6 \neq 0$. So the only homomorphism of complexes $T_j \to T_5[1]$, $j \neq 5$, is the zero map.

It follows that T is a tilting complex for A_{16}, and by Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(A_{16})}(T)$ is derived equivalent to A_{16}. Since we want to show that E is isomorphic to A_6, we use the alternating sum formula of Proposition 2.14 and compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$ which is the Cartan matrix of A_6 up to permutation.

Now we have to define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver of A_6.



First we have the embedding $\mathrm{id} : T_4 \to T_5$ (in degree zero). Moreover, we have the homomorphisms $\alpha_2\alpha_3 : T_5 \to T_2$ and $\alpha_4\alpha_5 : T_2 \to T_4$. Finally, we also have homomorphisms $\alpha_1, \alpha_2, \alpha_3, \alpha_6$ and α_7 as before.

Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $\alpha_7\alpha_3$, $\alpha_3\alpha_6$, $\alpha_3\alpha_4\alpha_5$ and $\alpha_4\alpha_5\alpha_2\alpha_3$ are zero since they were zero in A_{16}. As we can see, the two paths from vertex 2 to vertex 4 are the same, i.e., we here have the correct commutativity-relation. There is also another commutativity-relation $\alpha_6\alpha_7 = \alpha_4\alpha_5\alpha_2$ between vertex 4 and 3 since these are the same paths in A_{16}. The path from vertex 5 to vertex 2 is the last zero-relation since the homomorphism $\alpha_4\alpha_5$ is homotopic to zero.

Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_6. From this we can conclude that $E \cong A_6$ and thus, A_6 and A_{16} are derived equivalent. Since A_{16}^{op} is sink/source equivalent to A_{19}, A_{19} is also derived equivalent to A_6^{op} which in turn is sink/source equivalent to A_{21}.

Hence, we get derived equivalences between all cluster-tilted algebras with associated polynomial $3(x^6 + x^3 + 1)$.

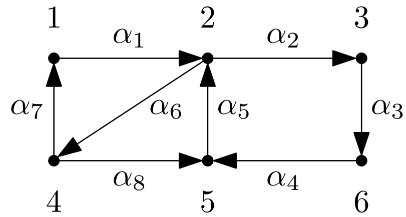
3.4 Derived equivalences for polynomial $4(x^6 + x^4 + x^2 + 1)$

We provide the following figure which displays what is proven in each subsection.

$$\begin{tikzcd} A_9 \arrow[r, dash, "3.4.1"] & A_5 \arrow[r, dash, "3.4.2"] \arrow[d, dash, "3.4.3"'] & A_{15} \arrow[r, dash, "3.4.2"] & A_{18} \arrow[r, dash, "3.4.1"] & A_{17} \\ & A_{13} & & & \end{tikzcd}$$

3.4.1 A_5 is derived equivalent to A_9

We consider A_5 with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex corresponding to the vertex 3, namely $T_i : 0 \to P_i \to 0$ for $i \in \{1, 2, 4, 5, 6\}$ are complexes concentrated in degree zero and $T_3 : 0 \to P_3 \xrightarrow{\alpha_2} P_2 \to 0$ is a complex in degrees -1 and 0.

Now we want to show that T is a tilting complex. We begin with possible maps $T_3 \to T_3[1]$ and $T_3 \to T_3[-1]$,

$$\begin{tikzcd}
 & 0 \arrow[r] & P_3 \arrow[r, "\alpha_2"] \arrow[d, "\alpha_2"] & P_2 \arrow[r] & 0 \\
0 \arrow[r] & P_3 \arrow[r, "\alpha_2"] & P_2 \arrow[r] \arrow[d, "0"] & 0 & \\
 & 0 \arrow[r] & P_3 \arrow[r, "\alpha_2"] & P_2 \arrow[r] & 0
\end{tikzcd}$$

Here α_2 is a basis of the space of homomorphisms between P_3 and P_2. But the homomorphism α_2 is homotopic to zero and in the second case there is no non-zero homomorphism $P_2 \to P_3$ (as we can see in the Cartan matrix of A_5).

Now consider possible maps $T_3 \to T_j[-1],\ j \neq 3$. These maps are given by a map of complexes as follows

$$\begin{tikzcd}
0 \arrow[r] & P_3 \arrow[r, "\alpha_2"] \arrow[d] & P_2 \arrow[r] & 0 \\
0 \arrow[r] & Q \arrow[r] & 0 &
\end{tikzcd}$$

where Q could be either P_1, P_2, P_4, P_5 or direct sums of these. Note that there is no non-zero homomorphism $P_3 \to P_6$ since this is a zero-relation in the quiver of A_5. There exist non-zero homomorphisms of complexes, but they are all homotopic to zero since every path from vertex $i \in \{1, 2, 4, 5\}$ to vertex 3 ends with α_2. Hence, every homomorphism from P_3 to P_1, P_2, P_4 or P_5 starts with α_2, up to scalars and thus, every homomorphism $P_3 \to Q$ can be factored through the map $\alpha_2 : P_3 \to P_2$. Directly from the definition we see that $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 2, 4, 5, 6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

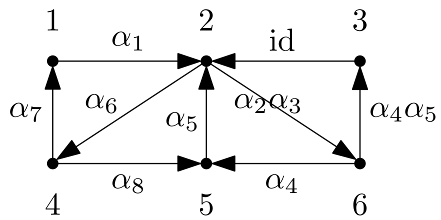
Finally, we have to consider maps $T_j \to T_3[1]$ for $j \neq 3$. These are given as follows

$$\begin{tikzcd}
0 \arrow[r] & Q \arrow[r] \arrow[d] & 0 & \\
0 \arrow[r] & P_3 \arrow[r, "\alpha_2"] & P_2 \arrow[r] & 0
\end{tikzcd}$$

where Q can be either P_5, P_6 or direct sums of these. Note that $\mathrm{Hom}(P_j, P_3) = 0$ for $j = 1,\ 2$ and $j = 4$. But no non-zero map can be zero when composed with α_2 since the path $\alpha_4\alpha_3\alpha_2 = \alpha_8\alpha_6 \neq 0$. So the only homomorphism of complexes $T_j \to T_3[1],\ j \neq 3$ is the zero map.

It follows that T is a tilting complex for A_5, and by Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(A_5)}(T)$ is derived equivalent to A_5. We want to show that E is isomorphic to A_9. Using the alternating sum formula of Proposition 2.14 we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$.

Now we have to define homomorphisms of complexes between the summands of T which correspond to the reversed arrows of the quiver of A_9 depicted below and show that these homomorphisms satisfy the defining relations of A_9, up to homotopy.



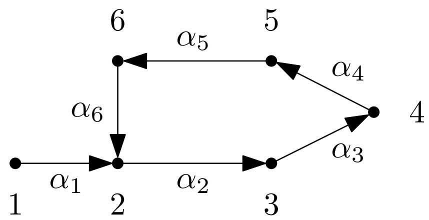
First we have the embedding id : $T_2 \to T_3$ (in degree zero). Moreover, we have the homomorphisms $\alpha_2\alpha_3 : T_6 \to T_2$ and $\alpha_4\alpha_5 : T_3 \to T_6$. Finally, we also have homomorphisms $\alpha_1, \alpha_4, \alpha_5, \alpha_6, \alpha_7$ and α_8 as before.

Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $\alpha_1\alpha_6$, $\alpha_6\alpha_7$, $\alpha_5\alpha_6$, $\alpha_5\alpha_2\alpha_3$ and $\alpha_2\alpha_3\alpha_4\alpha_5$ are zero since they were zero in A_5. As we can see, the two paths from vertex 4 to vertex 2 and the two paths from vertex 2 to vertex 5 are the same, since we have the same commutativity-relations in A_5. It is easy to see that the two paths from vertex 6 to vertex 2 are also the same. The last zero-relation $\alpha_2\alpha_3$ between vertex 6 and 3 is given by the homomorphism from T_3 to T_2 in degree zero. This is indeed a zero-relation since the homomorphism $\alpha_2\alpha_3$ is homotopic to zero.

Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_9. We have shown that they satisfy the defining relations of A_9 and that the Cartan matrices of E and A_9 coincide. From this we can conclude that $E \cong A_9$ and thus, A_9 and A_5 are derived equivalent. Since A_{17} is the opposite algebra of A_9, A_{17} is derived equivalent to $A_5^{\mathrm{op}} = A_{18}$.

3.4.2 A_{15} is derived equivalent to A_5 and A_{18}

We consider A_{15} with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex corresponding to the vertex 2, that is $T_i : 0 \to P_i \to 0$ for $i \in \{1,3,4,5,6\}$ are complexes concentrated in degree zero and $T_2 : 0 \to P_2 \stackrel{(\alpha_1,\alpha_6)}{\longrightarrow} P_1 \oplus P_6 \to 0$ in degrees -1 and 0.

Now we want to show that T is a tilting complex. We begin with possible maps $T_2 \to T_2[1]$ and $T_2 \to T_2[-1]$,

$$\begin{array}{ccccccccc}
 & & 0 & \to & P_2 & \stackrel{(\alpha_1,\alpha_6)}{\longrightarrow} & P_1 \oplus P_6 & \to & 0 \\
 & & & & \downarrow \psi & & & & \\
0 & \to & P_2 & \stackrel{(\alpha_1,\alpha_6)}{\longrightarrow} & P_1 \oplus P_6 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_2 & \stackrel{(\alpha_1,\alpha_6)}{\longrightarrow} & P_1 \oplus P_6 & \to & 0
\end{array}$$

where $\psi \in \mathrm{Hom}(P_2, P_1 \oplus P_6)$ and $(\alpha_1, 0)$, $(0, \alpha_6)$ is a basis of this two-dimensional space. The first homomorphism is homotopic to zero (as we can easily see). In the second case there is no non-zero homomorphism $P_1 \oplus P_6 \to P_2$ (as we can see in the Cartan matrix of A_{15}).

Now consider possible maps $T_2 \to T_j[-1]$, $j \neq 2$. These maps are given by a map of complexes as follows

$$\begin{array}{ccccccc}
0 & \to & P_2 & \stackrel{(\alpha_1,\alpha_6)}{\longrightarrow} & P_1 \oplus P_6 & \to & 0 \\
 & & \downarrow & & & & \\
0 & \to & Q & \to & 0 & &
\end{array}$$

where Q could be either P_1, P_4, P_5, P_6 or direct sums of these. Note that there is no non-zero homomorphism $P_2 \to P_3$ since this is a zero-relation in the quiver of A_{15}. There exist non-zero homomorphisms

of complexes, but they are all homotopic to zero since every path from vertex $i \in \{1,4,5,6\}$ to vertex 2 ends with α_1 or α_6. Thus, every homomorphism from P_2 to P_1, P_4, P_5 or P_6 starts with α_1 or α_6, up to scalars. Hence, every homomorphism $P_2 \to Q$ can be factored through the map $(\alpha_1, \alpha_6) : P_2 \to P_1 \oplus P_6$. Directly from the definition we see that $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1,3,4,5,6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

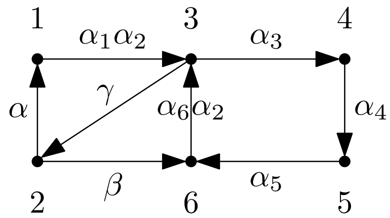
Finally, we have to consider maps $T_j \to T_2[1]$ for $j \neq 2$. These are given as follows

$$\begin{array}{ccccccc} 0 & \to & Q & \to & 0 & & \\ & & \downarrow & & & & \\ 0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_6)} & P_1 \oplus P_6 & \to & 0 \end{array}$$

where Q can be either P_3, P_4, P_5 or direct sums of these. Note that $\mathrm{Hom}(P_j, P_2) = 0$ for $j = 1$ and $j = 6$. But no non-zero map can be zero when composed with both α_1 and α_6 since the path $\alpha_2\alpha_1$ is not a zero-relation. So the only homomorphism of complexes $T_j \to T_2[1]$, $j \neq 2$, is the zero map.

It follows that T is a tilting complex for A_{15}, and by Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(A_{15})}(T)$ is derived equivalent to A_{15}. We want to show that E is isomorphic to A_5. Using the alternating sum formula of Proposition 2.14 we can compute the Cartan matrix of E to be $\begin{pmatrix} 1&0&1&1&1&0 \\ 1&1&1&1&0&1 \\ 0&1&1&1&1&1 \\ 0&0&0&1&1&1 \\ 0&0&1&0&1&1 \\ 0&0&1&1&0&1 \end{pmatrix}$ which is the Cartan matrix of A_5 up to permutation.

Now we have to define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver of A_5 depicted below and show that these homomorphisms satisfy the defining relations of A_5, up to homotopy.



First we have the embeddings $\alpha := (\mathrm{id}, 0) : T_1 \to T_2$ and $\beta := (0, \mathrm{id}) : T_6 \to T_2$ (in degree zero). Then we define $\gamma : T_2 \to T_3$ by the map $(0, \alpha_3\alpha_4\alpha_5) : P_1 \oplus P_6 \to P_3$ in degree 0. This is a homomorphism of complexes since $\alpha_2\alpha_3\alpha_4\alpha_5 = 0$ in A_{15}. Moreover, we have the homomorphisms $\alpha_1\alpha_2 : T_3 \to T_1$ and $\alpha_6\alpha_2 : T_3 \to T_6$. Finally, we also have homomorphisms α_3, α_4 and α_5 as before.

Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $\alpha_6\alpha_2\alpha_3\alpha_4$, $\alpha_4\alpha_5\alpha_6\alpha_2$ and $\alpha_5\alpha_6\alpha_2\alpha_3$ in the 4-cycle are zero since they were zero in A_{15}. As we can see, the two paths from vertex 3 to vertex 6 are the same, i.e., we here have the right commutativity-relation. There is also another commutativity-relation $\alpha\alpha_1\alpha_2 = \beta\alpha_6\alpha_2$ between vertex 2 and 3 which is given by the two homomorphisms from T_3 to the first and second summand of T_2. These are indeed the same paths since the homomorphism $(\alpha_2\alpha_1, 0)$ is homotopic to $(0, \alpha_2\alpha_6)$. Because $\alpha_2\alpha_3\alpha_4\alpha_5 = 0$ the paths from vertex 6 to vertex 2 and from vertex 1 to 2 are zero in E. The last zero-relation is given by the concatenation of α and γ.

Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_5. We have shown that they satisfy the defining relations of A_5 and that the Cartan matrices of E and A_5 coincide. From this we can conclude that $E \cong A_5$ and thus, A_{15} and A_5 are derived equivalent. Since A_{18} is the opposite algebra of A_5, A_{18} is derived equivalent to A_{15}^{op} and since A_{15} is sink/source equivalent to A_{15}^{op} we get derived equivalences between A_5, A_{15} and A_{18}. With the above result, we have derived equivalences between A_5, A_9, A_{15}, A_{17} and A_{18}.

3.4.3 A_{13} is derived equivalent to A_5

Consider the algebra A_{13} with the following quiver

$$\begin{tikzcd}
\overset{1}{\bullet} \arrow[r, "\alpha_1"] & \overset{2}{\bullet} \arrow[d, "\alpha_2"] & \overset{3}{\bullet} \arrow[l, "\alpha_7"'] \\
\underset{4}{\bullet} \arrow[u, "\alpha_4"] & \underset{5}{\bullet} \arrow[l, "\alpha_3"] \arrow[r, "\alpha_5"'] & \underset{6}{\bullet} \arrow[u, "\alpha_6"']
\end{tikzcd}$$

Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex corresponding to the vertex 4. Explicitly, $T_i : 0 \to P_i \to 0$ for $i \in \{1, 2, 3, 5, 6\}$ are complexes concentrated in degree zero and $T_4 : 0 \to P_4 \xrightarrow{\alpha_3} P_5 \to 0$ in degrees -1 and 0.

Now we want to show that T is a tilting complex. We begin with possible maps $T_4 \to T_4[1]$ and $T_4 \to T_4[-1]$,

$$\begin{array}{cccccccc}
 & & 0 & \to & P_4 & \xrightarrow{\alpha_3} & P_5 & \to & 0 \\
 & & & & \downarrow \alpha_3 & & & & \\
0 & \to & P_4 & \xrightarrow{\alpha_3} & P_5 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_4 & \xrightarrow{\alpha_3} & P_5 & \to & 0
\end{array}$$

Here α_3 is a basis of the space of homomorphisms between P_4 and P_5. The first homomorphism is homotopic to zero (as we can easily see). In the second case there is no non-zero homomorphism $P_5 \to P_4$ (as we can see in Cartan matrix of A_{13}).

Now consider possible maps $T_4 \to T_j[-1]$, $j \neq 4$. These maps are given by a map of complexes as follows

$$\begin{array}{ccccccc}
0 & \to & P_4 & \xrightarrow{\alpha_3} & P_5 & \to & 0 \\
 & & \downarrow & & & & \\
0 & \to & Q & \to & 0 & &
\end{array}$$

where Q could be either P_2, P_3, P_5 or direct sums of these. Note that there is no non-zero homomorphism $P_4 \to P_1$ and $P_4 \to P_6$ since these are zero-relation in the quiver of A_{13}. There exist non-zero homomorphisms of complexes, but they are all homotopic to zero since every homomorphism from P_4 to P_2, P_3 or P_5 starts with α_3, up to scalars. Thus, every homomorphism $P_4 \to Q$ can be factored through the map $\alpha_3 : P_4 \to P_5$. Hence, $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 2, 3, 5, 6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

Finally, we have to consider maps $T_j \to T_4[1]$ for $j \neq 4$. These are given as follows

$$\begin{array}{ccccccc}
0 & \to & Q & \to & 0 & & \\
 & & \downarrow & & & & \\
0 & \to & P_4 & \xrightarrow{\alpha_3} & P_5 & \to & 0
\end{array}$$

where Q can be either P_1, P_2 or direct sums of these since $\mathrm{Hom}(P_j, P_4) = 0$ for $j = 3,\ 5$ and $j = 6$. But no non-zero map can be zero when composed with α_3 since the path $\alpha_1\alpha_4\alpha_3 = \alpha_7\alpha_6\alpha_5 \neq 0$. So the only homomorphism of complexes $T_j \to T_4[1]$, $j \neq 4$, is the zero map.

It follows that T is a tilting complex for A_{13}, and by Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(A_{13})}(T)$ is derived equivalent to A_{13}. We claim that E is isomorphic to A_5 and we use the alternating sum formula of Proposition 2.14 for computing the Cartan matrix of E which is given as follows $\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$ and which is the Cartan matrix of A_5 up to permutation.

Now we define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver of A_5, depicted below.

$$\begin{tikzcd}
4 \arrow[r, "\mathrm{id}"] & 5 \arrow[r, "\alpha_5"] \arrow[dl, "\alpha_3\alpha_4"'] & 6 \arrow[d, "\alpha_6"] \\
1 \arrow[u, "\alpha_1\alpha_2"] \arrow[r, "\alpha_1"'] & 2 \arrow[u, "\alpha_2"'] & 3 \arrow[l, "\alpha_7"]
\end{tikzcd}$$

First we have the embedding $\mathrm{id} : T_5 \to T_4$ (in degree zero). Moreover, we have the homomorphisms $\alpha_1\alpha_2 : T_4 \to T_1$ and $\alpha_3\alpha_4 : T_1 \to T_5$. Finally, we also have homomorphisms $\alpha_1, \alpha_2, \alpha_5, \alpha_6$ and α_7 as before.

Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $\alpha_6\alpha_7\alpha_2$, $\alpha_7\alpha_2\alpha_5$, $\alpha_2\alpha_5\alpha_6$, $\alpha_2\alpha_3\alpha_4$, $\alpha_3\alpha_4\alpha_1$ and thus $\alpha_3\alpha_4\alpha_1\alpha_2$ are zero since they were zero in A_{13}. As we can see, the two paths $\alpha_5\alpha_6\alpha_7$ and $\alpha_3\alpha_4\alpha_1$ from vertex 5 to vertex 2 are the same since we have the same commutativity-relation in A_{13}. It is easy to see that the two path from vertex 1 to vertex 5 are also the same. The last zero-relation $\alpha_3\alpha_4$ between vertex 4 and 1 is given by the homomorphism from T_1 to T_4 in degree zero. This is indeed a zero-relation since the homomorphism $\alpha_3\alpha_4$ is homotopic to zero.

Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_5. We have shown that they satisfy the defining relations of A_5 and that the Cartan matrices of E and A_5 coincide. From this we can conclude that $E \cong A_5$ and thus, A_{13} and A_5 are derived equivalent. Hence, we get derived equivalences between A_5, A_9, A_{13}, A_{15}, A_{17} and A_{18}.

Therefore, we have shown that all cluster-tilted algebras with associated polynomial $4(x^6+x^4+x^2+1)$ are derived equivalent.

A Cluster-tilted algebras of type E_7

First we list all quivers of the cluster-tilted algebras of type E_7. Algebras with the same polynomial associated with their Cartan matrix are grouped in one table. According to Theorem 1.1, these groups turn out to be the derived equivalence classes.

Note that a tuple (a, b) stands for an arrow $a \to b$ and that the numbering of the algebras in the tables results from the numbering of the whole list.

$\boldsymbol{x^7 - x^6 + x^4 - x^3 + x - 1}$	
algebra KQ/I	quiver Q
A_1	$(1,2),(2,3),(3,4),(4,5),(4,7),(5,6)$

$\boldsymbol{2(x^7 - x^5 + 2x^4 - 2x^3 + x^2 - 1)}$	
algebra KQ/I	quiver Q
A_2	$(1,2),(2,3),(3,4),(4,5),(4,7),(5,6),(6,4)$
A_{13}	$(1,2),(2,3),(3,4),(4,5),(4,7),(5,3),(5,6),(6,4)$
A_{20}	$(1,2),(2,3),(3,4),(4,5),(5,3),(5,6),(5,7),(6,4)$

$\boldsymbol{2(x^7 - x^5 + x^4 - x^3 + x^2 - 1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_3	$(2,1),(3,2),(3,4),$ $(5,3),(5,6),(6,7),(7,5)$	A_4	$(2,1),(3,2),(3,4),$ $(3,5),(5,6),(6,3),(7,6)$
A_5	$(2,1),(3,2),(3,4),$ $(3,7),(4,5),(5,3),(6,4)$	A_{12}	$(2,1),(2,3),(3,4),(4,2),$ $(4,5),(5,3),(6,4),(7,6)$
A_{16}	$(1,2),(2,5),(3,2),(3,6),$ $(4,2),(5,3),(5,4),(7,5)$	A_{25}	$(1,2),(2,3),(3,5),(4,3),$ $(5,4),(5,6),(6,3),(6,7)$

$\boldsymbol{2(x^7 - 2x^5 + 4x^4 - 4x^3 + 2x^2 - 1)}$	
algebra KQ/I	quiver Q
A_{18}	$(1,2),(2,3),(3,4),(4,5),(5,3),(5,6),(6,4),(6,7)$

$\boldsymbol{3(x^7 - 1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_6	$(1,2),(2,3),(3,4),$ $(4,5),(4,7),(5,6),(6,3)$	A_7	$(1,2),(2,3),(3,4),$ $(4,5),(5,6),(6,3),(6,7)$
A_8	$(1,2),(2,3),(3,4),$ $(3,7),(4,5),(5,2),(6,4)$	A_{17}	$(1,2),(2,3),(3,4),(3,7)$ $(4,5),(5,6),(6,3),(7,6)$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{19}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,3),(6,7),(7,5)$	A_{21}	$(1,2),(2,3),(3,4),(3,7),$ $(4,2),(4,5),(5,6),(6,3)$
A_{23}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,3),(7,4)$	A_{26}	$(1,2),(2,3),(3,4),(4,5),$ $(4,7),(5,3),(6,4),(7,2)$
A_{27}	$(1,2),(2,3),(3,4),(4,2),$ $(4,5),(5,6),(6,3),(6,7)$	A_{28}	$(1,2),(2,4),(3,2),(4,3),$ $(4,6),(5,2),(6,5),(7,6)$
A_{29}	$(2,1),(2,3),(3,4),(4,5)$ $(5,2),(5,6),(6,4),(7,6)$	A_{36}	$(1,2),(2,3),(3,4),(4,5),$ $(4,7),(5,6),(6,3),(7,3)$
A_{37}	$(2,1),(2,3),(3,4),(3,5),$ $(4,2),(5,6),(6,2),(7,3)$	A_{39}	$(1,2),(2,3),(3,4),(4,2),$ $(4,5),(4,7),(5,6),(6,3)$
A_{44}	$(1,2),(2,3),(3,4),(3,7),$ $(4,5),(5,3),(5,6),(6,7),(7,5)$	A_{47}	$(2,1),(2,3),(3,4),(3,5)$ $(4,2),(5,2),(5,6),(6,3),(7,4)$
A_{51}	$(2,1),(2,3),(2,5),(3,4),$ $(4,2),(5,4),(5,6),(6,2),(7,6)$	A_{52}	$(1,2),(2,3),(3,4),(3,6)$ $(4,5),(5,3),(6,5),(6,7),(7,3)$
A_{54}	$(1,2),(2,3),(3,4),(4,2),$ $(4,5),(5,3),(5,6),(5,7),(6,4)$	A_{56}	$(1,2),(2,3),(3,5),(4,3),$ $(5,4),(5,6),(6,3),(6,7),(7,5)$
A_{59}	$(2,1),(2,3),(3,4),(3,5),$ $(4,2),(5,2),(5,6),(6,3),(6,7)$	A_{60}	$(1,2),(2,3),(2,7),(3,1),$ $(3,4),(4,5),(5,2),(5,6),(6,4)$
A_{66}	$(1,2),(2,3),(3,4),(4,2),$ $(4,5),(4,7),(5,3),(5,6),(6,4)$	A_{67}	$(1,2),(2,3),(2,6),(3,4),$ $(4,2),(4,5),(5,3),(6,4),(7,4)$
A_{72}	$(2,1),(2,3),(2,6),(3,4)$ $(3,7),(4,2),(4,5),(5,6),(6,4)$	A_{73}	$(2,1),(2,3),(2,7),(3,4),$ $(4,5),(4,6),(5,3),(6,2),(7,6)$
A_{75}	$(1,2),(2,3),(2,5),(3,4),$ $(4,2),(5,4),(5,6),(6,2),(7,4)$	A_{86}	$(1,2),(2,3),(2,7),(3,4),(3,6),$ $(4,5),(5,3),(6,2),(6,5),(7,6)$
A_{87}	$(1,2),(2,3),(3,4),(3,5),(4,6)$ $(5,2),(5,6),(6,3),(6,7),(7,4)$	A_{89}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,4),(6,7),(7,5)$
A_{97}	$(1,2),(2,5),(3,2),(3,6),(4,2),$ $(5,3),(5,4),(6,5),(6,7),(7,3)$		

$\mathbf{4(x^7+x^6-x^5+x^4-x^3+x^2-x-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{14}	$(2,1),(2,3),(3,4),(4,2),$ $(5,2),(5,6),(6,7),(7,5)$	A_{15}	$(2,1),(2,3),(2,5),(3,4),$ $(4,2),(5,6),(6,2),(7,6)$
A_{22}	$(1,2),(2,3),(3,4),(4,2),$ $(4,5),(4,7),(5,6),(6,4)$	A_{31}	$(2,1),(2,3),(2,5),(3,4),$ $(4,2),(5,6),(6,7),(7,5)$
A_{46}	$(2,1),(2,3),(3,4),(3,5),$ $(4,2),(5,2),(5,6),(6,7),(7,5)$	A_{57}	$(2,1),(2,3),(2,5),(3,4),$ $(4,2),(5,4),(5,6),(6,7),(7,5)$

$\mathbf{4(x^7+x^6-x^5-x^4+x^3+x^2-x-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{45}	$(1,2),(2,3),(3,4),(4,2),$ $(4,5),(4,6),(5,3),(6,7),(7,4)$	A_{50}	$(2,1),(2,3),(3,4),(3,6),$ $(4,2),(4,5),(5,3),(6,7),(7,3)$

$\mathbf{4(x^7+x^6-2x^5+2x^4-2x^3+2x^2-x-1)}$	
algebra KQ/I	quiver Q
A_{53}	$(2,1),(2,3),(3,4),(4,2),(4,5),(5,3),(5,6),(6,7),(7,5)$

$\mathbf{4(x^7+x^5-x^4+x^3-x^2-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{9}	$(1,2),(2,3),(3,4),$ $(4,5),(5,6),(6,7),(7,3)$	A_{10}	$(1,2),(2,3),(3,4),$ $(4,5),(5,6),(6,2),(7,4)$
A_{30}	$(2,1),(2,4),(3,2),(4,3),$ $(4,5),(5,6),(6,7),(7,2)$	A_{33}	$(2,1),(2,3),(3,4),(4,5),$ $(5,2),(5,6),(6,7),(7,4)$
A_{34}	$(1,2),(2,3),(3,4),(4,5),$ $(4,6),(5,2),(6,7),(7,3)$	A_{40}	$(2,1),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,2),(7,4)$
A_{43}	$(2,1),(2,3),(2,7),(3,4),$ $(4,5),(5,6),(6,2),(7,6)$	A_{48}	$(1,2),(2,3),(2,7),(3,4),$ $(4,2),(4,5),(5,6),(6,7),(7,4)$
A_{58}	$(2,1),(2,3),(3,4),(3,5),$ $(4,2),(5,6),(5,7),(6,2),(7,3)$	A_{61}	$(1,2),(2,3),(3,4),(4,2),$ $(4,5),(5,3),(5,6),(6,7),(7,4)$
A_{63}	$(1,2),(2,3),(3,4),(4,5),$ $(4,6),(5,2),(6,3),(6,7),(7,4)$	A_{64}	$(1,2),(2,3),(3,4),(4,5),$ $(4,7),(5,2),(5,6),(6,4),(7,6)$
A_{68}	$(1,2),(2,3),(2,6),(2,7),$ $(3,1),(3,4),(4,5),(5,2),(6,5)$	A_{69}	$(1,2),(2,3),(3,4),(3,6),$ $(4,2),(4,5),(5,3),(6,7),(7,5)$
A_{70}	$(1,2),(2,3),(3,4),(3,7),$ $(4,5),(4,6),(5,2),(6,3),(7,6)$	A_{76}	$(2,1),(2,3),(2,4),(3,6),$ $(4,5),(5,6),(6,2),(6,7),(7,5)$
A_{77}	$(1,2),(1,4),(2,6),(3,2),$ $(4,5),(5,1),(6,5),(6,7),(7,3)$	A_{78}	$(1,2),(2,5),(3,2),(3,7),$ $(4,3),(5,6),(6.3),(7,4),(7,6)$
A_{80}	$(1,2),(2,6),(3,2),(3,4),$ $(4,5),(5,6),(6,3),(6,7),(7,2)$	A_{85}	$(1,2),(2,3),(2,4),(3,5),(4,5),$ $(5,2),(5,6),(6,4),(6,7),(7,5)$
A_{88}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,7),(5,3),(5,6),(6,4),(7,6)$	A_{91}	$(1,2),(2,3),(3,4),(3,6),(4,2)$ $(4,5),(5,3),(6,5),(6,7),(7,3)$
A_{92}	$(1,2),(2,5),(3,2),(3,7),(4,3),$ $(5,1),(5,6),(6,3),(7,4),(7,6)$	A_{99}	$(2,1),(2,5),(3,2),(3,4),(4,5),$ $(5,3),(5,6),(5,7),(6,4),(7,2)$
A_{100}	$(1,5),(2,1),(2,6),(3,2),(3,4),$ $(4,7),(5,2),(6,5),(6,7),(7,3)$	A_{101}	$(1,2),(2,3),(2,5),(3,6),(4,1),$ $(5,4),(5,6),(6,2),(6,7),(7,3)$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{103}	$(1,2),(2,6),(3,2),(3,7),(4,3),$ $(5,1),(6,3),(6,5),(7,4),(7,6)$	A_{109}	$(1,2),(1,4),(2,3),(2,5),(3,6),$ $(4,5),(5,1),(5,6),(6,2),(6,7),(7,3)$
A_{110}	$(1,4),(2,1),(2,3),(2,5),(3,6),$ $(4,2),(5,4),(5,6),(6,2),(6,7),(7,3)$	A_{111}	$(1,2),(2,3),(2,4),(3,5),(4,1),$ $(4,5),(5,2),(5,6),(6,3),(6,7),(7,5)$

$\mathbf{4(x^7+x^5-2x^4+2x^3-x^2-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{38}	$(2,1),(2,3),(3,4),(4,5),$ $(4,7),(5,6),(6,2),(7,3)$	A_{41}	$(2,1),(2,3),(3,4),(4,5),$ $(5,6),(6,2),(6,7),(7,5)$
A_{71}	$(2,1),(2,3),(3,4),(4,5),$ $(4,6),(5,3),(6,2),(6,7),(7,4)$	A_{95}	$(1,6),(2,1),(3,2),(3,7),(4,3),$ $(5,1),(6,3),(6,5),(7,4),(7,6)$
A_{98}	$(1,2),(2,3),(2,5),(3,7),(4,3),$ $(5,1),(5,6),(6,7),(7,2),(7,4)$		

$\mathbf{5(x^7+x^5-x^4+x^3-x^2-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{11}	$(2,1),(2,3),(3,4),$ $(4,5),(5,6),(6,7),(7,2)$	A_{42}	$(1,2),(2,3),(3,4),(4,1)$ $(4,5),(5,6),(6,7),(7,3)$
A_{65}	$(1,2),(1,7),(2,3),(3,1),$ $(3,4),(4,5),(5,6),(6,7),(7,3)$	A_{79}	$(1,2),(2,3),(3,4),(4,1)$ $(4,5),(5,6),(5,7),(6,3),(7,4)$
A_{81}	$(1,2),(2,3),(3,4),(3,7),$ $(4,1),(4,5),(5,3),(6,5),(7,6)$	A_{82}	$(1,2),(2,3),(3,4),(3,7),$ $(4,1),(4,5),(5,6),(6,3),(7,6)$
A_{83}	$(1,2),(2,3),(3,4),(3,7),$ $(4,5),(4,6),(5,1),(6,3),(7,6)$	A_{90}	$(1,2),(2,5),(3,2),(3,6),(4,1),$ $(4,7),(5,3),(5,4),(6,5),(7,5)$
A_{94}	$(2,1),(2,3),(2,6),(3,4),(4,2),$ $(4,5),(5,6),(6,4),(6,7),(7,2)$	A_{102}	$(1,5),(2,1),(2,3),(3,6),(4,3),$ $(4,7),(5,6),(6,2),(6,4),(7,6)$
A_{105}	$(1,3),(2,1),(2,4),(2,7),(3,2)$ $(4,5),(5,2),(6,5),(7,3),(7,6)$	A_{106}	$(1,2),(2,3),(3,1),(3,4),(3,5),$ $(4,7),(5,2),(5,6),(6,3),(7,6)$
A_{107}	$(1,3),(2,1),(2,6),(3,2),(3,7)$ $(4,3),(5,2),(6,3),(6,5),(7,4),(7,6)$	A_{108}	$(1,7),(2,1),(2,3),(2,6),(3,4),$ $(4,2),(4,5),(5,6),(6,4),(6,7),(7,2)$
A_{112}	$(1,2),(1,6),(2,3),(3,1),(3,4),$ $(3,5),(4,2),(5,6),(5,7),(6,3),(7,3)$		

$\mathbf{6(x^7+x^6-x^4+x^3-x-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{24}	$(2,1),(2,3),(3,4),(4,5),$ $(5,2),(5,6),(6,7),(7,5)$	A_{32}	$(2,1),(2,3),(3,4),(3,6),$ $(4,5),(5,2),(6,7),(7,3)$
A_{49}	$(1,2),(2,3),(3,1),(3,4),$ $(3,6),(4,5),(5,2),(6,7),(7,3)$	A_{55}	$(1,2),(2,3),(3,1),(3,4),$ $(4,5),(5,6),(6,3),(6,7),(7,5)$
A_{62}	$(1,2),(2,3),(3,1),(3,4),$ $(4,5),(5,6),(5,7),(6,3),(7,4)$	A_{74}	$(1,2),(2,3),(2,4),(3,1),$ $(4,5),(4,6),(5,2),(6,7),(7,2)$
A_{84}	$(1,2),(2,3),(3,1),(3,4),(3,6),$ $(4,5),(5,3),(5,7),(6,5),(7,6)$	A_{93}	$(1,5),(2,1),(2,3),(3,5),(4,1),$ $(5,2),(5,4),(5,7),(6,5),(7,6)$
A_{96}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(4,6),(5,3),(6,3),(6,7),(7,4)$		

$\mathbf{6(x^7+x^5-x^2-1)}$	
algebra KQ/I	quiver Q
A_{104}	$(1,2),(2,3),(2,5),(3,6),(4,1),(5,4),(5,6),(6,2),(6,7),(7,5)$

$\mathbf{8(x^7+x^6+x^5-x^4+x^3-x^2-x-1)}$	
algebra KQ/I	quiver Q
A_{35}	$(1,2),(2,3),(3,1),(3,4),(4,5),(5,6),(6,7),(7,3)$

B Derived equivalences for cluster-tilted algebras of type E_7

First we list the opposite algebra for each cluster-tilted algebra. By a result of Rickard [18, Prop. 9.1], if A is derived equivalent to B, also A^{op} is derived equivalent to B^{op}.

After this, we list the cluster-tilted algebra, the corresponding tilting complex, the derived equivalent cluster-tilted algebra with permutation of the vertices (up to sink/source equivalence) and the resulting equivalence for the opposite algebras (if necessary).

The tilting complexes are all of the form introduced in Section 3.1, arising from vertices and resulting in good mutations. If we have a tilting complex $T = \bigoplus_{i=1}^{7} T_i$ with $T_i : 0 \to P_i \to 0,\ i \in \{1,3,4,5,6,7\}$ (in degree zero) and $T_2 : 0 \to P_2 \to P_1 \oplus P_5 \to 0$ in degrees -1 and 0 we write $(2;1,5)$ for T_2 and know that the other summands are just the stalk complexes.

We write the permutation as a product of disjoint cycles. If we have a permutation (135)(67) the

labeling of the vertices changes as follows: $1 \to 3$, $3 \to 5$, $5 \to 1$, $6 \to 7$, $7 \to 6$ and the labeling of the other vertices is left unchanged.

B.1 Polynomial $2(x^7 - x^5 + 2x^4 - 2x^3 + x^2 - 1)$

$A_2^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_2$, $A_{13}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{20}$

$A_{13}(*)$	$(4;3,6,7)$	$\underset{\text{der}}{\sim}$	A_2	(567)	$\Rightarrow$	$A_{20} \underset{\text{der}}{\sim} A_2$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.2 Polynomial $2(x^7 - x^5 + x^4 - x^3 + x^2 - 1)$

$A_3^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_3$, $A_4^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_5$, $A_{12}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{25}$, $A_{16}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{16}$

A_5	$(4;3,6)$	$\underset{\text{der}}{\sim}$	A_3	(457)	$\Rightarrow$	$A_4 \underset{\text{der}}{\sim} A_3$
A_{16}	$(2;1,3,4)$	$\underset{\text{der}}{\sim}$	A_4	$(156)(23)$	$\Rightarrow$	$A_{16} \underset{\text{der}}{\sim} A_5$
A_{25}	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_5	(16247)	$\Rightarrow$	$A_{12} \underset{\text{der}}{\sim} A_4$

B.3 Polynomial $3(x^7 - 1)$

$A_6^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_7$, $A_8^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_8$, $A_{17}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{36}$, $A_{19}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{23}$, $A_{21}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{39}$, $A_{26}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{28}$, $A_{27}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{29}$, $A_{37}^{\mathrm{op}} = A_{37}$, $A_{44}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{56}$, $A_{47}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{72}$, $A_{51}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{66}$, $A_{52}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{52}$, $A_{54}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{59}$, $A_{60}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{73}$, $A_{67}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{75}$, $A_{86}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{97}$, $A_{87}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{89}$

A_6	$(3;2,6)$	$\underset{\text{der}}{\sim}$	A_{51}	$(17)(264)(35)$	$\Rightarrow$	$A_7 \underset{\text{der}}{\sim} A_{66}$
$A_6(*)$	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{56}	(46)	$\Rightarrow$	$A_7 \underset{\text{der}}{\sim} A_{44}$
A_8	$(4;3,6)$	$\underset{\text{der}}{\sim}$	A_{47}	$(17)(2354)$	$\Rightarrow$	$A_8 \underset{\text{der}}{\sim} A_{72}$
A_8	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{66}	$(16)(2435)$	$\Rightarrow$	$A_8 \underset{\text{der}}{\sim} A_{51}$
$A_8(*)$	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{75}	$(167)(24)(35)$	$\Rightarrow$	$A_8 \underset{\text{der}}{\sim} A_{67}$
A_{17}	$(3;2,6)$	$\underset{\text{der}}{\sim}$	A_{86}	(3456)	$\Rightarrow$	$A_{36} \underset{\text{der}}{\sim} A_{97}$
A_{17}	$(4;3)$	$\underset{\text{der}}{\sim}$	A_{52}	$(47)(56)$	$\Rightarrow$	$A_{36} \underset{\text{der}}{\sim} A_{52}$
A_{19}	$(3;2,6)$	$\underset{\text{der}}{\sim}$	A_{87}	(345)	$\Rightarrow$	$A_{23} \underset{\text{der}}{\sim} A_{89}$
A_{23}	$(6;5)$	$\underset{\text{der}}{\sim}$	A_{44}	(467)	$\Rightarrow$	$A_{19} \underset{\text{der}}{\sim} A_{56}$
A_{44}	$(6;5)$	$\underset{\text{der}}{\sim}$	A_{17}	$(47)(56)$	$\Rightarrow$	$A_{56} \underset{\text{der}}{\sim} A_{36}$
A_{51}	$(3;2)$	$\underset{\text{der}}{\sim}$	A_{21}	$(17)(236)(45)$	$\Rightarrow$	$A_{66} \underset{\text{der}}{\sim} A_{39}$
A_{54}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{73}	(176425)	$\Rightarrow$	$A_{59} \underset{\text{der}}{\sim} A_{60}$
A_{66}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{60}	$(16)(2534)$	$\Rightarrow$	$A_{51} \underset{\text{der}}{\sim} A_{73}$
A_{67}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{37}	$(17)(23564)$	$\Rightarrow$	$A_{75} \underset{\text{der}}{\sim} A_{37}$
A_{75}	$(3;2)$	$\underset{\text{der}}{\sim}$	A_{28}	(3546)	$\Rightarrow$	$A_{67} \underset{\text{der}}{\sim} A_{26}$
A_{87}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{27}	(456)	$\Rightarrow$	$A_{89} \underset{\text{der}}{\sim} A_{29}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.4 Polynomial $4(x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x - 1)$

$A_{14}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{31}$, $A_{15}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{22}$, $A_{46}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{57}$

A_{15}	$(6;5,7)$	$\underset{\text{der}}{\sim}$	A_{22}	$(1735)(246)$		
A_{31}	$(5;2,7)$	$\underset{\text{der}}{\sim}$	A_{15}	(56)	$\Rightarrow$	$A_{14} \underset{\text{der}}{\sim} A_{22}$
$A_{46}(*)$	$(2;1,4,5)$	$\underset{\text{der}}{\sim}$	A_{31}	(134)	$\Rightarrow$	$A_{57} \underset{\text{der}}{\sim} A_{14}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.5 Polynomial $4(x^7+x^6-x^5-x^4+x^3+x^2-x-1)$

$A_{45}^{\mathrm{op}} = A_{50}$

A_{50}	$(3;2,5,7)$	$\underset{\text{der}}{\sim}$	A_{45}	(3476)

B.6 Polynomial $4(x^7+x^5-x^4+x^3-x^2-1)$

$A_9^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_9$, $A_{10}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{10}$, $A_{30}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{43}$, $A_{33}^{\mathrm{op}} = A_{34}$, $A_{40}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{40}$, $A_{48}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{80}$, $A_{58}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{76}$, $A_{61}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{69}$, $A_{63}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{64}$, $A_{68}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{68}$, $A_{70}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{78}$, $A_{77}^{\mathrm{op}} = A_{77}$, $A_{85}^{\mathrm{op}} = A_{99}$, $A_{88}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{91}$, $A_{92}^{\mathrm{op}} = A_{100}$, $A_{101}^{\mathrm{op}} = A_{103}$, $A_{109}^{\mathrm{op}} = A_{109}$, $A_{110}^{\mathrm{op}} = A_{111}$

A_9	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{69}	$(34)(576)$	$\Rightarrow$	$A_9 \underset{\text{der}}{\sim} A_{61}$
A_{10}	$(2;1,6)$	$\underset{\text{der}}{\sim}$	A_{78}	(1724)	$\Rightarrow$	$A_{10} \underset{\text{der}}{\sim} A_{70}$
A_{10}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{63}	(456)	$\Rightarrow$	$A_{10} \underset{\text{der}}{\sim} A_{64}$
$A_{30}(*)$	$(2;1,3,7)$	$\underset{\text{der}}{\sim}$	A_{48}	(13)	$\Rightarrow$	$A_{43} \underset{\text{der}}{\sim} A_{80}$
A_{30}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{58}	$(34)(567)$	$\Rightarrow$	$A_{43} \underset{\text{der}}{\sim} A_{76}$
$A_{33}(*)$	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{103}	$(1724)(56)$	$\Rightarrow$	$A_{34} \underset{\text{der}}{\sim} A_{101}$
A_{33}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{88}	$(45)(67)$	$\Rightarrow$	$A_{34} \underset{\text{der}}{\sim} A_{91}$
A_{33}	$(6;5)$	$\underset{\text{der}}{\sim}$	A_{78}	$(35)(46)$	$\Rightarrow$	$A_{34} \underset{\text{der}}{\sim} A_{70}$
A_{34}	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{99}	(475)	$\Rightarrow$	$A_{33} \underset{\text{der}}{\sim} A_{85}$
A_{34}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{48}	(3567)	$\Rightarrow$	$A_{33} \underset{\text{der}}{\sim} A_{80}$
$A_{40}(*)$	$(2;1,6)$	$\underset{\text{der}}{\sim}$	A_{92}	(1724)	$\Rightarrow$	$A_{40} \underset{\text{der}}{\sim} A_{100}$
A_{68}	$(6;2)$	$\underset{\text{der}}{\sim}$	A_{30}	(176543)	$\Rightarrow$	$A_{68} \underset{\text{der}}{\sim} A_{43}$
A_{77}	$(2;1,3)$	$\underset{\text{der}}{\sim}$	A_{110}	(1743526)	$\Rightarrow$	$A_{77} \underset{\text{der}}{\sim} A_{111}$
A_{78}	$(2;1,3)$	$\underset{\text{der}}{\sim}$	A_{111}	(247635)	$\Rightarrow$	$A_{70} \underset{\text{der}}{\sim} A_{110}$
A_{88}	$(7;4)$	$\underset{\text{der}}{\sim}$	A_{61}	(1)	$\Rightarrow$	$A_{91} \underset{\text{der}}{\sim} A_{69}$
A_{103}	$(2;1,3)$	$\underset{\text{der}}{\sim}$	A_{100}	$(15)(26)(37)$	$\Rightarrow$	$A_{101} \underset{\text{der}}{\sim} A_{92}$
A_{109}	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{63}	$(17456)(23)$	$\Rightarrow$	$A_{109} \underset{\text{der}}{\sim} A_{64}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.7 Polynomial $4(x^7+x^5-2x^4+2x^3-x^2-1)$

$A_{38}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{41}$, $A_{71}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{71}$, $A_{95}^{\mathrm{op}} = A_{98}$

A_{38}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{71}	(57)	$\Rightarrow$	$A_{41} \underset{\text{der}}{\sim} A_{71}$
$A_{41}(*)$	$(2;1,6)$	$\underset{\text{der}}{\sim}$	A_{95}	(15724)	$\Rightarrow$	$A_{38} \underset{\text{der}}{\sim} A_{98}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.8 Polynomial $5(x^7+x^5-x^4+x^3-x^2-1)$

$A_{11}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{11}$, $A_{42}^{\mathrm{op}} = A_{42}$, $A_{65}^{\mathrm{op}} = A_{83}$, $A_{79}^{\mathrm{op}} = A_{82}$, $A_{81}^{\mathrm{op}} = A_{81}$, $A_{90}^{\mathrm{op}} = A_{105}$, $A_{94}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{94}$, $A_{102}^{\mathrm{op}} = A_{106}$, $A_{107}^{\mathrm{op}} = A_{108}$, $A_{112}^{\mathrm{op}} = A_{112}$

$A_{11}(*)$	$(2;1,7)$	$\underset{\text{der}}{\sim}$	A_{65}	(12)	$\Rightarrow$	$A_{11} \underset{\text{der}}{\sim} A_{83}$
A_{79}	$(3;2,6)$	$\underset{\text{der}}{\sim}$	A_{112}	(143)	$\Rightarrow$	$A_{82} \underset{\text{der}}{\sim} A_{112}$
A_{83}	$(7;3)$	$\underset{\text{der}}{\sim}$	A_{42}	$(16)(27)$	$\Rightarrow$	$A_{65} \underset{\text{der}}{\sim} A_{42}$
$A_{94}(*)$	$(2;1,4,7)$	$\underset{\text{der}}{\sim}$	A_{106}	$(175)(23)$	$\Rightarrow$	$A_{94} \underset{\text{der}}{\sim} A_{102}$
A_{106}	$(1;3)$	$\underset{\text{der}}{\sim}$	A_{81}	$(12)(4567)$	$\Rightarrow$	$A_{102} \underset{\text{der}}{\sim} A_{81}$
A_{106}	$(4;3)$	$\underset{\text{der}}{\sim}$	A_{112}	(1574)	$\Rightarrow$	$A_{102} \underset{\text{der}}{\sim} A_{112}$
A_{105}	$(1;2)$	$\underset{\text{der}}{\sim}$	A_{82}	$(123)(47)(56)$	$\Rightarrow$	$A_{90} \underset{\text{der}}{\sim} A_{79}$
A_{105}	$(4;2)$	$\underset{\text{der}}{\sim}$	A_{83}	(1742365)	$\Rightarrow$	$A_{90} \underset{\text{der}}{\sim} A_{65}$

A_{108}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{105}	(37654)	$\Rightarrow$	$A_{107} \underset{\text{der}}{\sim} A_{90}$	

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.9 Polynomial $6(x^7 + x^6 - x^4 + x^3 - x - 1)$

$A_{24}^{\text{op}} \underset{\text{s/s}}{\sim} A_{32}$, $A_{49}^{\text{op}} = A_{74}$, $A_{55}^{\text{op}} = A_{62}$, $A_{84}^{\text{op}} = A_{96}$, $A_{93}^{\text{op}} = A_{93}$

$A_{24}(*)$	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{96}	$(1726)(345)$	$\Rightarrow$	$A_{32} \underset{\text{der}}{\sim} A_{84}$
$A_{32}(*)$	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{93}	$(135)(67)$	$\Rightarrow$	$A_{24} \underset{\text{der}}{\sim} A_{93}$
A_{49}	$(4;3)$	$\underset{\text{der}}{\sim}$	A_{93}	$(14352)(67)$	$\Rightarrow$	$A_{74} \underset{\text{der}}{\sim} A_{93}$
A_{96}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{55}	(45)	$\Rightarrow$	$A_{84} \underset{\text{der}}{\sim} A_{62}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

C Cluster-tilted algebras of type E_8

$\boldsymbol{x^8 - x^7 + x^5 - x^4 + x^3 - x + 1}$	
algebra KQ/I	quiver Q
A_1	$(1,2),(2,3),(4,3),(5,4),(6,5),(7,6),(8,3)$

$\boldsymbol{2(x^8 - x^6 + 2x^5 - 2x^4 + 2x^3 - x^2 + 1)}$	
algebra KQ/I	quiver Q
A_2	$(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,5),(8,5)$
A_{19}	$(1,2),(2,3),(3,4),(4,5),(5,6),(6,4),(6,7),(6,8),(7,5)$
A_{28}	$(1,2),(2,3),(3,4),(4,5),(5,6),(5,8),(6,4),(6,7),(7,5)$

$\boldsymbol{2(x^8 - x^6 + x^5 + x^3 - x^2 + 1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_3	$(1,2),(2,3),(3,4),(4,5),$ $(5,3),(6,4),(7,4),(8,7)$	A_4	$(1,2),(2,3),(4,3),(5,3),$ $(5,6),(6,7),(7,5),(8,7)$
A_5	$(1,2),(2,3),(4,3),(5,3),$ $(6,5),(6,7),(7,8),(8,6)$	A_6	$(1,2),(2,3),(3,5),(4,3),$ $(5,6),(5,8),(6,3),(7,6)$
A_7	$(1,2),(2,3),(4,3),(5,3),$ $(5,6),(6,7),(6,8),(7,5)$	A_{10}	$(1,2),(2,3),(3,5),(4,3),$ $(5,6),(5,7),(6,3),(7,8)$
A_{23}	$(1,2),(2,3),(4,3),(4,5),$ $(4,7),(5,6),(6,4),(7,6),(8,7)$	A_{31}	$(1,2),(3,2),(3,4),(3,6),$ $(4,5),(5,3),(5,8),(6,5),(7,4)$
A_{35}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,4),(7,4),(7,8)$	A_{46}	$(2,1),(3,2),(3,4),(4,5),$ $(4,6),(4,8),(5,3),(6,3),(7,5)$

$\boldsymbol{2(x^8 - 2x^6 + 4x^5 - 4x^4 + 4x^3 - 2x^2 + 1)}$	
algebra KQ/I	quiver Q
A_{25}	$(1,2),(2,3),(3,4),(4,5),(5,6),(6,4),(6,7),(7,5),(8,7)$

$\boldsymbol{3(x^8 + x^4 + 1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_8	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(7,4),(7,8)$	A_9	$(1,2),(2,3),(4,3),(4,5),$ $(5,6),(6,7),(7,4),(8,5)$
A_{12}	$(1,2),(3,2),(3,4),(4,5),$ $(5,6),(5,8),(6,3),(7,4)$	A_{14}	$(2,1),(3,2),(3,4),(4,5),$ $(5,6),(5,8),(6,3),(7,6)$
A_{17}	$(2,1),(2,3),(3,4),(4,5),$ $(5,2),(5,6),(7,3),(8,4)$	A_{26}	$(1,2),(2,3),(3,4),(4,5),$ $(5,3),(5,6),(6,7),(6,8),(7,4)$
A_{30}	$(1,2),(2,3),(4,3),(4,5),$ $(5,6),(6,7),(7,4),(7,8),(8,6)$	A_{33}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(6,8),(7,4),(8,5)$
A_{34}	$(1,2),(2,3),(3,4),(4,5),$ $(5,3),(5,6),(6,7),(7,4),(8,7)$	A_{43}	$(1,2),(2,3),(3,4),(4,5),$ $(4,7),(5,6),(6,4),(7,8),(8,6)$
A_{44}	$(1,2),(2,3),(3,4),(4,5),$ $(4,8),(5,3),(5,6),(6,7),(7,4)$	A_{47}	$(1,2),(3,2),(3,4),(4,5),$ $(5,6),(5,8),(6,3),(6,7),(7,5)$
A_{53}	$(1,2),(2,3),(3,4),(3,7),$ $(4,5),(5,6),(6,3),(7,6),(8,5)$	A_{60}	$(1,2),(2,3),(4,3),(4,5),$ $(5,6),(5,8),(6,7),(7,4),(8,4)$
A_{61}	$(2,1),(3,2),(3,4),(4,5),$ $(4,6),(5,3),(6,7),(6,8),(7,3)$	A_{66}	$(1,2),(2,3),(3,4),(4,5),$ $(5,3),(5,6),(5,8),(6,7),(7,4)$
A_{67}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,3),(7,4),(8,5)$	A_{76}	$(2,1),(2,3),(3,4),(3,6),$ $(4,5),(5,2),(6,2),(7,3),(8,7)$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{80}	$(1,2),(2,3),(3,4),(4,5),$ $(4,7),(5,6),(6,3),(7,3),(8,4)$	A_{84}	$(1,2),(2,3),(3,4),(3,8),$ $(4,2),(4,5),(5,6),(6,3),(7,4)$
A_{92}	$(1,2),(2,3),(3,4),(4,5),(4,8),$ $(5,6),(6,4),(6,7),(7,5),(8,6)$	A_{94}	$(1,2),(2,3),(3,4),(4,5),(5,3),$ $(5,6),(6,4),(6,7),(7,5),(8,6)$
A_{100}	$(1,2),(2,3),(3,4),(4,5),(4,8),$ $(5,3),(5,6),(6,4),(6,7),(8,6)$	A_{102}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,3),(5,6),(6,4),(6,8),(8,5)$
A_{109}	$(1,2),(2,3),(4,3),(4,5),(5,6),$ $(5,7),(6,4),(7,4),(7,8),(8,5)$	A_{110}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,4),(6,7),(8,5)$
A_{111}	$(1,2),(2,3),(2,5),(3,4),(4,2),$ $(5,6),(6,4),(6,7),(7,5),(8,7)$	A_{123}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,6),(6,4),(7,6),(7,8),(8,4)$
A_{131}	$(1,2),(2,3),(3,4),(3,5),(5,2),$ $(5,6),(6,3),(6,7),(7,5),(7,8)$	A_{132}	$(1,2),(2,3),(3,4),(3,5),(5,2),$ $(5,6),(6,7),(7,3),(7,8),(8,6)$
A_{144}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(4,8),(6,3),(6,7),(7,4),(8,7)$	A_{148}	$(2,1),(3,2),(3,4),(4,5),(4,7),$ $(5,3),(5,6),(6,4),(7,6),(8,5)$
A_{149}	$(2,1),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(6,3),(6,7),(7,5),(8,4)$	A_{154}	$(1,2),(3,2),(3,4),(4,5),(4,6),$ $(4,8),(5,3),(6,3),(6,7),(7,4)$
A_{163}	$(1,2),(2,3),(3,4),(3,8),(4,5),$ $(5,3),(5,6),(5,7),(7,8),(8,5)$	A_{169}	$(1,2),(2,3),(3,4),(3,7),(4,5),$ $(4,6),(5,3),(6,3),(7,6),(8,4)$
A_{171}	$(2,1),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(5,8),(6,3),(6,7),(7,5)$	A_{173}	$(1,2),(2,3),(3,4),(3,5),(3,7),$ $(5,2),(5,6),(6,3),(7,6),(8,7)$
A_{187}	$(1,2),(3,2),(3,4),(4,5),(5,3),$ $(5,6),(5,7),(7,4),(7,8),(8,5)$	A_{196}	$(2,1),(2,3),(3,4),(3,6),(3,7),$ $(4,2),(4,5),(5,3),(6,2),(7,8)$
A_{206}	$(2,1),(2,3),(2,5),(3,4),(4,2),$ $(5,4),(5,6),(6,2),(7,5),(8,6)$	A_{218}	$(2,1),(2,3),(3,4),(3,8),(4,2),$ $(4,5),(4,6),(6,3),(6,7),(7,4)$
A_{221}	$(1,2),(2,3),(2,4),(4,1),(4,5),$ $(5,6),(6,2),(6,7),(7,5),(8,5)$	A_{222}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,2),(5,6),(5,7),(7,4),(8,3)$
A_{232}	$(1,2),(2,3),(3,4),(4,5),(5,3),$ $(5,6),(6,4),(6,7),(7,5),(7,8),(8,6)$	A_{242}	$(1,2),(3,2),(3,4),(4,5),(4,8),$ $(5,3),(5,6),(6,4),(6,7),(7,8),(8,6)$
A_{247}	$(1,2),(2,3),(3,4),(3,7),(4,5),$ $(5,3),(5,6),(6,7),(7,5),(7,8),(8,6)$	A_{272}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,4),(6,7),(7,5),(8,6)$
A_{275}	$(2,1),(2,3),(3,4),(3,7),(4,2),$ $(4,5),(5,3),(5,6),(6,7),(7,5),(7,8)$	A_{277}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,3),(6,3),(6,7),(7,4),(7,8),(8,6)$
A_{305}	$(2,1),(2,3),(2,6),(3,4),(4,2),$ $(4,5),(5,6),(6,4),(6,7),(7,5),(8,5)$		

$\mathbf{4(x^8+x^7-x^6+x^5+x^3-x^2+x+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{20}	$(1,2),(2,3),(3,4),(4,5),$ $(4,7),(5,3),(6,4),(7,8),(8,4)$	A_{21}	$(1,2),(2,3),(3,1),(4,2),$ $(5,2),(5,6),(6,7),(7,5),(8,7)$
A_{22}	$(1,2),(2,3),(3,1),(4,2),$ $(5,2),(6,5),(6,7),(7,8),(8,6)$	A_{27}	$(2,1),(2,3),(3,4),(4,2),$ $(4,6),(5,4),(6,7),(7,4),(8,3)$
A_{29}	$(2,1),(2,3),(3,4),(4,2),$ $(4,5),(5,7),(6,5),(7,8),(8,5)$	A_{36}	$(1,2),(2,3),(3,4),(4,2),$ $(5,3),(5,6),(5,7),(7,8),(8,5)$
A_{37}	$(1,2),(2,3),(2,4),(2,5),$ $(3,1),(6,5),(6,7),(7,8),(8,6)$	A_{41}	$(2,1),(3,2),(3,4),(4,5),$ $(5,3),(5,7),(6,5),(7,8),(8,5)$
A_{49}	$(1,2),(2,3),(2,4),(2,5),$ $(3,1),(5,6),(6,7),(6,8),(7,5)$	A_{52}	$(1,2),(2,3),(3,1),(4,2),$ $(5,4),(5,7),(6,5),(7,8),(8,5)$
A_{89}	$(2,1),(2,3),(2,5),(3,4),(4,2),$ $(5,4),(6,5),(6,7),(7,8),(8,6)$	A_{90}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(6,3),(6,7),(6,8),(7,5)$
A_{98}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(6,4),(6,7),(6,8),(7,5)$	A_{105}	$(1,2),(2,3),(2,4),(3,1),(4,5),$ $(5,6),(5,8),(6,4),(6,7),(7,5)$
A_{106}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,2),(5,6),(6,7),(7,5),(8,7)$	A_{122}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,2),(5,6),(6,7),(7,8),(8,6)$
A_{124}	$(2,1),(2,3),(2,5),(3,4),(4,2),$ $(5,4),(5,6),(6,7),(7,8),(8,6)$	A_{142}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(5,7),(7,4),(7,8),(8,5)$

$\mathbf{4(x^8+x^7-x^6+2x^4-x^2+x+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{24}	$(1,2),(2,3),(3,5),(4,3),$ $(5,6),(6,3),(6,7),(7,8),(8,6)$	A_{32}	$(1,2),(2,3),(3,5),(4,3),$ $(5,6),(5,7),(6,3),(7,8),(8,5)$
A_{93}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,6),(5,3),(6,7),(7,8),(8,6)$	A_{107}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,6),(5,3),(6,7),(7,4),(7,8)$
A_{113}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(4,6),(4,7),(5,3),(6,3),(8,6)$	A_{120}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(5,7),(6,3),(7,3),(7,8)$
A_{121}	$(1,2),(2,3),(2,4),(3,1),(4,5),$ $(5,6),(5,7),(6,4),(7,4),(7,8)$	A_{137}	$(2,1),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(6,4),(6,7),(7,8),(8,6)$
A_{146}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,6),(5,3),(6,7),(6,8),(7,4)$	A_{152}	$(1,2),(2,3),(3,1),(3,4),(3,7),$ $(4,5),(5,3),(5,6),(7,5),(8,4)$
A_{153}	$(1,2),(2,3),(3,4),(3,6),(4,2),$ $(4,5),(5,3),(6,7),(6,8),(8,3)$	A_{155}	$(1,2),(2,3),(3,4),(3,6),(4,2),$ $(4,5),(5,3),(6,7),(7,8),(8,6)$

$\mathbf{4(x^8+x^7-2x^6+2x^5+2x^3-2x^2+x+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{95}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,6),(6,4),(6,7),(7,5),(8,7)$	A_{96}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(6,3),(6,7),(7,5),(8,7)$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{116}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(6,4),(6,7),(7,5),(8,7)$	A_{119}	$(2,1),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,7),(7,8),(8,6)$

$\boldsymbol{4(x^8+x^6-x^5+2x^4-x^3+x^2+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{11}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(7,8),(8,4)$	A_{13}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(7,3),(8,5)$
A_{16}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(6,8),(7,3)$	A_{40}	$(2,1),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,2),(7,4),(8,7)$
A_{42}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,3),(6,7),(7,8),(8,5)$	A_{54}	$(2,1),(2,3),(3,4),(4,5),$ $(4,6),(5,2),(6,7),(7,3),(8,6)$
A_{55}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,3),(7,8),(8,4)$	A_{58}	$(1,2),(2,3),(3,4),(3,8),$ $(4,2),(4,5),(5,6),(6,7),(7,3)$
A_{72}	$(1,2),(2,3),(3,4),(4,2),$ $(4,5),(4,6),(6,7),(7,8),(8,3)$	A_{85}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,8),(6,2),(6,7),(7,5)$
A_{87}	$(1,2),(2,3),(2,7),(3,4),$ $(4,5),(5,6),(5,8),(6,2),(7,6)$	A_{99}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(5,7),(5,8),(6,3),(7,4)$
A_{103}	$(1,2),(2,3),(3,4),(4,5),(5,3),$ $(5,6),(6,4),(6,7),(7,8),(8,5)$	A_{104}	$(1,2),(2,3),(3,4),(3,8),(4,5),$ $(5,3),(5,6),(6,7),(7,4),(8,5)$
A_{112}	$(1,2),(2,3),(3,4),(4,5),(5,6),$ $(5,7),(6,3),(7,4),(7,8),(8,5)$	A_{126}	$(1,2),(2,3),(3,4),(3,8),(4,2),$ $(4,5),(4,6),(6,7),(7,3),(8,7)$
A_{127}	$(2,1),(2,3),(2,8),(3,4),(4,5),$ $(5,6),(5,7),(6,2),(7,4),(8,6)$	A_{128}	$(1,2),(2,3),(3,4),(3,6),(4,5),$ $(5,3),(5,8),(6,2),(6,7),(7,5)$
A_{129}	$(1,2),(2,3),(3,4),(4,5),(5,6),$ $(5,8),(6,3),(6,7),(7,5),(8,7)$	A_{133}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,7),(6,8),(7,4)$
A_{134}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(6,3),(6,7),(7,8),(8,5)$	A_{135}	$(1,2),(2,3),(3,4),(4,5),(5,6),$ $(6,3),(6,7),(7,5),(7,8),(8,6)$
A_{136}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,3),(6,3),(6,7),(7,8),(8,4)$	A_{143}	$(1,2),(2,3),(3,4),(3,5),(5,2),$ $(5,6),(6,7),(6,8),(7,3),(8,5)$
A_{150}	$(2,1),(2,3),(3,4),(3,6),(4,2),$ $(4,5),(5,3),(6,7),(7,5),(8,7)$	A_{151}	$(1,2),(2,3),(2,5),(3,4),(4,2),$ $(5,6),(6,7),(6,8),(7,4),(8,5)$
A_{162}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,3),(5,6),(6,4),(7,8),(8,6)$	A_{170}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,2),(6,7),(7,3),(7,8),(8,6)$
A_{172}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(6,7),(7,2),(7,8),(8,6)$	A_{175}	$(2,1),(2,3),(3,4),(4,5),(5,2),$ $(5,6),(6,7),(7,4),(7,8),(8,6)$
A_{176}	$(1,2),(2,3),(3,4),(4,5),(5,2),$ $(5,6),(6,4),(6,7),(6,8),(7,5)$	A_{177}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,2),(5,6),(6,7),(7,3),(7,8)$
A_{182}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,6),(6,4),(7,6),(7,8),(8,3)$	A_{185}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(6,7),(6,8),(7,2),(8,5)$
A_{186}	$(1,2),(2,3),(3,4),(3,7),(3,8),$ $(4,2),(4,5),(5,6),(6,3),(7,6)$	A_{192}	$(1,2),(2,3),(3,4),(3,6),(4,2),$ $(4,5),(5,3),(6,7),(7,5),(8,6)$
A_{193}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,6),(6,7),(6,8),(7,4),(8,3)$	A_{207}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,7),(7,4),(8,7)$
A_{208}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,3),(6,2),(6,7),(7,4),(8,4)$	A_{224}	$(1,2),(2,3),(3,4),(4,5),(5,3),$ $(5,6),(5,8),(6,4),(6,7),(7,5),(8,7)$
A_{225}	$(1,2),(2,3),(3,4),(3,8),(4,5),$ $(5,3),(5,6),(6,4),(6,7),(7,5),(8,5)$	A_{226}	$(2,1),(2,3),(3,4),(3,5),(3,7),$ $(4,2),(5,2),(5,6),(6,3),(7,6),(8,7)$
A_{227}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(5,8),(6,3),(6,7),(7,5),(8,7)$	A_{231}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,3),(5,6),(6,4),(7,6),(7,8),(8,4)$
A_{237}	$(1,2),(2,3),(2,8),(3,4),(4,2),$ $(4,5),(5,3),(5,6),(6,4),(6,7),(8,4)$	A_{238}	$(1,2),(2,3),(2,8),(3,4),(4,2),$ $(4,5),(5,6),(6,3),(6,7),(7,5),(8,4)$
A_{239}	$(2,1),(3,2),(3,4),(4,5),(4,7),$ $(4,8),(5,3),(5,6),(6,4),(7,6),(8,3)$	A_{240}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,2),(5,6),(6,7),(7,3),(7,8),(8,6)$
A_{243}	$(1,2),(2,3),(3,4),(4,1),(4,5),$ $(5,3),(5,6),(6,4),(6,7),(7,5),(8,7)$	A_{258}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(6,3),(6,7),(7,5),(7,8),(8,6)$
A_{273}	$(1,2),(2,3),(3,4),(4,5),(4,8),$ $(5,2),(5,6),(6,4),(6,7),(7,8),(8,6)$	A_{276}	$(2,1),(2,3),(3,4),(4,5),(4,8),$ $(5,3),(5,6),(6,4),(6,7),(7,5),(8,2)$
A_{282}	$(1,2),(2,3),(2,7),(3,4),(4,2),$ $(4,5),(5,6),(6,7),(7,4),(7,8),(8,6)$	A_{286}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,4),(6,7),(7,8),(8,5)$
A_{304}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,2),(5,6),(6,7),(6,8),(7,5),(8,3)$	A_{311}	$(1,2),(2,3),(2,8),(3,4),(4,2),$ $(4,5),(5,6),(6,4),(6,7),(7,5),(8,6)$
A_{324}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(5,7),(6,2),(7,3),(7,8),(8,5)$	A_{333}	$(1,2),(2,3),(3,4),(3,6),(4,2),(4,5),$ $(5,3),(6,5),(6,7),(7,3),(7,8),(8,6)$
A_{337}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,3),$ $(5,6),(6,4),(6,7),(7,5),(7,8),(8,6)$	A_{338}	$(1,2),(2,3),(3,4),(3,7),(4,2),(4,5),$ $(5,3),(5,6),(6,7),(7,5),(7,8),(8,6)$
A_{342}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,3),$ $(5,6),(5,8),(6,7),(7,5),(8,4),(8,7)$	A_{343}	$(1,2),(2,3),(2,5),(3,1),(3,4),(4,2),$ $(5,1),(5,6),(6,2),(6,7),(7,5),(8,7)$
A_{352}	$(1,2),(2,3),(3,1),(3,4),(4,5),(5,2),$ $(5,6),(6,4),(6,7),(7,5),(7,8),(8,6)$	A_{361}	$(1,2),(2,3),(3,4),(3,7),(3,8),(4,2),$ $(4,5),(5,3),(5,6),(6,7),(7,5),(8,2)$
A_{363}	$(1,2),(2,3),(3,1),(3,4),(4,2),(4,5),$ $(4,8),(5,6),(5,7),(7,4),(8,3),(8,7)$	A_{366}	$(2,1),(2,3),(2,8),(3,4),(4,2),(4,5),$ $(5,6),(5,8),(6,4),(6,7),(7,5),(8,4)$
A_{370}	$(1,2),(2,3),(3,1),(3,4),(4,5),(4,8),$ $(5,2),(5,6),(6,4),(6,7),(7,8),(8,6)$	A_{388}	$(1,2),(2,3),(2,4),(3,1),(4,1),(4,5),$ $(5,2),(5,6),(6,4),(6,7),(7,5),(7,8),(8,6)$

$\boldsymbol{4(x^8+x^6-2x^5+4x^4-2x^3+x^2+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{48}	$(1,2),(2,3),(3,4),(4,5),$	A_{70}	$(1,2),(2,3),(3,4),(4,2),$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
	$(4,7),(5,6),(6,2),(7,3),(8,7)$		$(4,5),(5,6),(5,8),(6,7),(7,3)$
A_{118}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,2),(5,6),(6,4),(7,3),(8,7)$	A_{160}	$(1,2),(2,3),(3,4),(3,7),(4,2),$ $(4,5),(5,6),(6,3),(7,6),(8,5)$
A_{161}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,6),(6,2),(6,7),(7,5),(8,4)$	A_{278}	$(1,2),(2,3),(2,8),(3,4),(3,7),$ $(4,1),(4,5),(5,3),(5,6),(7,2),(8,7)$
A_{302}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(5,7),(6,3),(7,4),(7,8),(8,5)$		

$\mathbf{5(x^8+x^6+x^4+x^2+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{18}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,8),(6,7),(7,2)$	A_{51}	$(2,1),(2,3),(3,4),(4,5),$ $(5,6),(5,8),(6,7),(7,2),(8,4)$
A_{65}	$(1,2),(2,3),(3,4),(4,5),$ $(4,8),(5,6),(6,7),(7,2),(8,3)$	A_{69}	$(2,1),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(6,8),(7,2),(8,5)$
A_{71}	$(2,1),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(7,2),(7,8),(8,6)$	A_{74}	$(1,2),(2,3),(3,4),(3,7),$ $(3,8),(4,5),(5,6),(6,1),(7,2)$
A_{77}	$(1,2),(2,3),(3,4),(4,5),$ $(4,6),(5,2),(6,7),(7,8),(8,3)$	A_{78}	$(1,2),(2,3),(2,8),(3,4),$ $(3,7),(4,5),(5,6),(6,1),(7,2)$
A_{83}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,2),(7,8),(8,4)$	A_{86}	$(2,1),(2,3),(3,4),(4,5),$ $(5,2),(5,6),(6,7),(7,8),(8,4)$
A_{125}	$(2,1),(2,3),(3,4),(4,5),(5,6),$ $(5,7),(6,2),(6,8),(7,4),(8,5)$	A_{140}	$(1,2),(2,3),(2,8),(3,4),(4,2),$ $(4,5),(5,6),(6,7),(7,8),(8,4)$
A_{159}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,6),(5,8),(6,2),(7,3),(8,4)$	A_{165}	$(2,1),(2,3),(2,7),(3,4),(4,2),$ $(4,5),(5,6),(6,4),(7,8),(8,6)$
A_{166}	$(2,1),(2,3),(3,4),(3,7),(4,5),$ $(4,8),(5,6),(6,2),(7,2),(8,3)$	A_{174}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,2),(5,6),(6,7),(7,8),(8,3)$
A_{178}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,6),(6,2),(6,7),(7,8),(8,5)$	A_{181}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,2),(6,7),(6,8),(7,4),(8,3)$
A_{183}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,2),(5,6),(6,4),(7,8),(8,6)$	A_{199}	$(1,2),(2,3),(3,4),(3,7),(4,5),$ $(5,6),(6,3),(7,2),(7,8),(8,6)$
A_{200}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,2),(6,3),(6,7),(7,8),(8,4)$	A_{201}	$(1,2),(2,3),(2,4),(2,8),(4,1),$ $(4,5),(5,6),(6,7),(7,2),(8,7)$
A_{202}	$(2,1),(2,3),(3,4),(4,5),(4,8),$ $(5,2),(5,6),(6,7),(7,4),(8,7)$	A_{203}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(5,7),(6,3),(7,8),(8,4)$
A_{204}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,6),(5,7),(6,2),(7,8),(8,4)$	A_{212}	$(1,2),(2,3),(2,8),(3,4),(4,2),$ $(4,5),(5,6),(6,7),(7,4),(8,7)$
A_{213}	$(1,2),(2,3),(3,4),(3,8),(4,5),$ $(4,6),(5,2),(6,7),(7,3),(8,7)$	A_{214}	$(1,2),(2,3),(3,4),(4,5),(4,8),$ $(5,6),(5,7),(6,2),(7,4),(8,7)$
A_{216}	$(1,2),(2,3),(3,4),(4,5),(5,6),$ $(5,7),(6,2),(7,4),(7,8),(8,5)$	A_{219}	$(1,2),(2,3),(3,4),(4,5),(5,2),$ $(5,6),(6,7),(6,8),(7,4),(8,5)$
A_{220}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(5,8),(6,7),(7,3),(8,2)$	A_{223}	$(1,2),(2,3),(3,4),(4,1),(4,5),$ $(5,6),(6,3),(6,7),(7,8),(8,5)$
A_{234}	$(1,2),(2,3),(3,4),(3,8),(4,5),$ $(5,3),(5,6),(5,7),(7,8),(8,2),(8,5)$	A_{241}	$(1,2),(1,4),(2,3),(3,1),(4,5),$ $(5,6),(5,8),(6,3),(6,7),(7,5),(8,7)$
A_{246}	$(1,2),(2,3),(2,8),(3,4),(4,2),$ $(4,5),(5,6),(5,7),(6,3),(7,4),(8,4)$	A_{249}	$(1,2),(2,3),(2,7),(3,1),(3,4),$ $(4,5),(5,2),(5,6),(6,4),(7,5),(8,3)$
A_{252}	$(1,2),(2,3),(3,4),(3,7),(4,2),$ $(4,5),(5,6),(6,3),(7,6),(7,8),(8,3)$	A_{260}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(5,7),(6,3),(7,2),(7,8),(8,5)$
A_{261}	$(1,2),(2,3),(3,4),(3,8),(4,2),$ $(4,5),(4,6),(6,3),(6,7),(7,4),(8,6)$	A_{262}	$(1,2),(2,3),(2,4),(2,5),(3,1),$ $(5,6),(5,7),(6,2),(7,1),(7,8),(8,5)$
A_{263}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,2),(5,6),(6,4),(7,6),(7,8),(8,4)$	A_{265}	$(2,1),(2,3),(3,4),(3,5),(3,8),$ $(4,2),(5,2),(5,6),(6,7),(7,3),(8,7)$
A_{266}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,8),(5,6),(5,7),(6,3),(7,4),(8,7)$	A_{267}	$(1,2),(1,4),(2,3),(3,1),(4,5),$ $(4,8),(5,6),(5,7),(6,3),(7,4),(8,7)$
A_{274}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,6),(5,7),(6,2),(7,4),(7,8),(8,5)$	A_{279}	$(1,2),(2,3),(3,4),(4,1),(4,5),$ $(5,3),(5,6),(6,7),(6,8),(7,4),(8,5)$
A_{281}	$(1,2),(1,8),(2,3),(3,1),(3,4),$ $(4,5),(5,6),(5,7),(6,4),(7,8),(8,3)$	A_{283}	$(1,2),(2,3),(3,4),(3,6),(4,1),$ $(4,5),(5,3),(6,5),(6,7),(7,3),(8,7)$
A_{285}	$(2,1),(2,3),(2,6),(3,4),(4,2),$ $(4,5),(5,6),(6,4),(6,7),(7,2),(8,7)$	A_{293}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(4,8),(5,2),(6,7),(7,4),(8,3),(8,7)$
A_{295}	$(1,2),(2,3),(2,7),(2,8),(3,1),$ $(3,4),(4,5),(5,2),(5,6),(6,4),(8,5)$	A_{296}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,7),(5,3),(5,6),(6,4),(7,8),(8,6)$
A_{297}	$(1,2),(2,3),(3,4),(4,1),(4,5),$ $(5,3),(5,6),(6,7),(7,4),(7,8),(8,6)$	A_{303}	$(1,2),(2,3),(3,4),(3,8),(4,1),$ $(4,5),(5,3),(5,6),(6,7),(7,5),(8,7)$
A_{306}	$(1,2),(2,3),(3,4),(3,8),(4,5),$ $(4,7),(5,1),(5,6),(6,4),(7,3),(8,7)$	A_{307}	$(1,2),(2,3),(2,4),(3,1),(4,5),$ $(4,7),(5,6),(6,1),(7,2),(7,8),(8,4)$
A_{310}	$(1,2),(1,5),(2,3),(3,4),(3,6),$ $(4,1),(5,4),(6,2),(6,7),(7,8),(8,3)$	A_{312}	$(1,2),(2,3),(3,4),(3,5),(3,7),$ $(4,2),(5,2),(5,6),(6,3),(7,8),(8,6)$
A_{314}	$(1,2),(2,3),(3,4),(3,6),(4,1),$ $(4,5),(5,3),(6,5),(6,7),(7,3),(8,6)$	A_{315}	$(1,2),(2,3),(3,1),(3,4),(4,2),$ $(4,5),(5,3),(5,6),(6,7),(7,8),(8,4)$
A_{318}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,1),(6,3),(6,7),(7,4),(7,8),(8,6)$	A_{321}	$(2,1),(2,3),(2,7),(3,4),(4,2),$ $(4,5),(5,6),(5,8),(6,4),(7,6),(8,4)$
A_{322}	$(1,2),(2,3),(3,4),(4,1),(4,5),$ $(5,6),(6,3),(6,7),(7,5),(7,8),(8,6)$	A_{326}	$(1,2),(2,3),(3,4),(3,8),(4,1),$ $(4,5),(5,3),(5,6),(6,7),(7,8),(8,5)$
A_{327}	$(1,2),(2,3),(3,4),(3,5),(3,7),$ $(4,2),(5,6),(6,3),(7,6),(7,8),(8,2)$	A_{328}	$(1,2),(2,3),(3,1),(3,4),(4,2),$ $(4,5),(5,6),(6,3),(6,7),(7,8),(8,5)$
A_{335}	$(1,2),(1,4),(2,3),(3,1),(3,6),(4,3),$ $(4,5),(5,6),(6,4),(6,7),(7,3),(8,7)$	A_{340}	$(1,2),(2,3),(3,1),(3,4),(3,8),(4,2),$ $(4,5),(5,3),(5,6),(6,4),(6,7),(8,5)$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{345}	(1, 2), (2, 3), (2, 6), (3, 4), (4, 2), (4, 5), (5, 6), (5, 8), (6, 4), (6, 7), (7, 5), (8, 4)	A_{346}	(1, 2), (2, 3), (2, 7), (3, 1), (3, 4), (4, 2), (4, 5), (5, 6), (5, 8), (6, 7), (7, 4), (8, 4)
A_{348}	(1, 2), (2, 3), (3, 4), (3, 8), (4, 2), (4, 5), (5, 3), (5, 6), (6, 4), (6, 7), (7, 5), (8, 5)	A_{349}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (4, 7), (5, 2), (5, 6), (6, 4), (7, 3), (7, 8), (8, 4)
A_{350}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (4, 8), (5, 2), (5, 6), (6, 4), (6, 7), (7, 5), (8, 3)	A_{351}	(1, 2), (2, 3), (2, 8), (3, 4), (4, 2), (4, 5), (4, 7), (5, 6), (6, 4), (7, 6), (7, 8), (8, 4)
A_{353}	(1, 2), (2, 3), (3, 4), (3, 5), (3, 7), (4, 2), (5, 2), (5, 6), (6, 3), (7, 6), (7, 8), (8, 3)	A_{354}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 2), (4, 5), (5, 3), (5, 6), (6, 7), (6, 8), (7, 4), (8, 5)
A_{355}	(2, 1), (2, 3), (3, 4), (3, 5), (3, 8), (4, 2), (5, 2), (5, 6), (6, 3), (6, 7), (7, 5), (8, 6)	A_{356}	(1, 2), (2, 3), (2, 6), (3, 1), (3, 4), (4, 2), (4, 5), (4, 7), (5, 6), (6, 4), (7, 3), (8, 7)
A_{357}	(1, 2), (2, 3), (2, 8), (3, 1), (3, 4), (4, 5), (4, 7), (5, 2), (5, 6), (6, 4), (7, 6), (8, 5)	A_{360}	(1, 2), (2, 3), (3, 1), (3, 4), (3, 8), (4, 5), (5, 3), (6, 2), (6, 7), (7, 8), (8, 5), (8, 6)
A_{362}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (4, 7), (5, 2), (5, 6), (6, 4), (7, 6), (7, 8), (8, 4)	A_{364}	(1, 2), (2, 3), (3, 4), (3, 7), (3, 8), (4, 1), (4, 5), (5, 3), (5, 6), (6, 7), (7, 5), (8, 2)
A_{365}	(1, 2), (2, 3), (3, 1), (3, 4), (3, 6), (4, 2), (4, 5), (5, 3), (6, 7), (7, 5), (7, 8), (8, 6)	A_{367}	(1, 2), (2, 3), (2, 5), (3, 1), (3, 4), (4, 2), (5, 4), (5, 6), (6, 7), (6, 8), (7, 2), (8, 5)
A_{368}	(1, 2), (2, 3), (3, 4), (3, 6), (3, 8), (4, 2), (4, 5), (5, 3), (6, 5), (6, 7), (7, 3), (8, 7)	A_{369}	(1, 2), (1, 8), (2, 3), (3, 1), (3, 4), (4, 5), (4, 8), (5, 6), (6, 7), (7, 4), (8, 3), (8, 7)
A_{371}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (4, 6), (5, 2), (6, 3), (6, 7), (7, 4), (7, 8), (8, 6)	A_{372}	(1, 2), (2, 3), (2, 5), (2, 7), (3, 1), (3, 4), (4, 2), (5, 1), (5, 6), (6, 2), (7, 6), (7, 8)
A_{373}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 1), (4, 5), (5, 6), (6, 2), (6, 7), (7, 5), (7, 8), (8, 6)	A_{374}	(1, 2), (2, 3), (3, 4), (3, 7), (4, 1), (4, 5), (5, 3), (5, 6), (6, 7), (6, 8), (7, 5), (8, 5)
A_{375}	(1, 2), (2, 3), (3, 1), (3, 4), (3, 8), (4, 2), (4, 5), (5, 3), (5, 6), (6, 7), (7, 5), (8, 7)	A_{378}	(1, 2), (2, 3), (3, 4), (3, 7), (4, 1), (4, 5), (5, 3), (5, 6), (6, 7), (7, 5), (7, 8), (8, 6)
A_{379}	(2, 1), (2, 3), (2, 6), (3, 4), (4, 2), (4, 5), (5, 6), (6, 4), (6, 7), (6, 8), (7, 5), (8, 2)	A_{380}	(1, 2), (2, 3), (2, 7), (3, 4), (4, 2), (4, 5), (5, 6), (5, 7), (6, 4), (7, 4), (7, 8), (8, 2)
A_{381}	(1, 2), (2, 3), (3, 1), (3, 4), (3, 8), (4, 2), (4, 5), (5, 3), (5, 6), (6, 7), (7, 8), (8, 5)	A_{382}	(1, 2), (2, 3), (2, 5), (3, 1), (3, 4), (4, 2), (5, 6), (6, 4), (6, 7), (7, 5), (7, 8), (8, 6)
A_{383}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 2), (4, 5), (5, 6), (5, 8), (6, 3), (6, 7), (7, 5), (8, 7)	A_{386}	(1, 2), (2, 3), (2, 6), (3, 1), (3, 4), (4, 2), (4, 5), (5, 6), (6, 4), (6, 7), (7, 5), (7, 8), (8, 6)
A_{389}	(1, 2), (2, 3), (3, 1), (3, 4), (3, 7), (4, 2), (4, 5), (5, 3), (5, 6), (6, 7), (6, 8), (7, 5), (8, 5)	A_{390}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 2), (4, 5), (5, 3), (5, 6), (5, 8), (6, 4), (6, 7), (7, 5), (8, 7)

$6(x^8 + x^6 + x^5 + x^3 + x^2 + 1)$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{15}	(2, 1), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 2)	A_{88}	(1, 2), (2, 3), (3, 4), (3, 5), (4, 1), (5, 6), (6, 7), (7, 8), (8, 2)
A_{179}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 2), (4, 5), (5, 6), (6, 7), (7, 8), (8, 3)	A_{184}	(1, 2), (2, 3), (2, 8), (3, 4), (4, 5), (5, 6), (6, 7), (7, 2), (8, 1), (8, 7)
A_{205}	(1, 2), (2, 3), (3, 4), (4, 5), (4, 8), (5, 1), (5, 6), (6, 7), (7, 4), (8, 3)	A_{209}	(1, 2), (2, 3), (3, 4), (3, 5), (4, 1), (5, 2), (5, 6), (6, 7), (7, 8), (8, 3)
A_{211}	(1, 2), (2, 3), (3, 4), (4, 5), (4, 7), (5, 1), (5, 6), (6, 4), (7, 8), (8, 3)	A_{215}	(1, 2), (2, 3), (3, 4), (3, 6), (4, 1), (4, 5), (5, 3), (6, 7), (7, 8), (8, 5)
A_{268}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (4, 6), (5, 3), (6, 2), (6, 7), (7, 8), (8, 4)	A_{270}	(1, 2), (2, 3), (3, 4), (4, 5), (4, 7), (4, 8), (5, 1), (5, 6), (6, 4), (7, 6), (8, 3)
A_{280}	(1, 2), (1, 7), (2, 3), (3, 1), (3, 4), (4, 5), (4, 8), (5, 6), (6, 7), (7, 3), (8, 3)	A_{290}	(1, 2), (2, 3), (3, 4), (3, 6), (4, 1), (4, 5), (5, 3), (5, 7), (6, 5), (7, 4), (8, 7)
A_{299}	(1, 2), (2, 3), (2, 7), (3, 1), (3, 4), (4, 5), (4, 6), (5, 2), (6, 3), (7, 8), (8, 5)	A_{300}	(1, 2), (2, 3), (3, 4), (3, 6), (3, 8), (4, 1), (4, 5), (5, 3), (6, 7), (7, 5), (8, 2)
A_{308}	(1, 2), (2, 3), (3, 4), (3, 6), (4, 1), (4, 5), (5, 3), (6, 7), (6, 8), (7, 5), (8, 3)	A_{309}	(1, 2), (2, 3), (3, 4), (3, 8), (4, 5), (4, 6), (5, 2), (6, 3), (6, 7), (7, 4), (8, 6)
A_{313}	(1, 2), (2, 3), (3, 4), (4, 5), (4, 7), (5, 1), (5, 6), (6, 4), (7, 6), (7, 8), (8, 4)	A_{317}	(1, 2), (2, 3), (3, 4), (4, 5), (4, 8), (5, 2), (5, 6), (6, 4), (6, 7), (7, 5), (8, 6)
A_{319}	(1, 2), (2, 3), (3, 4), (3, 8), (4, 5), (5, 3), (5, 6), (6, 7), (7, 8), (8, 2), (8, 5)	A_{320}	(1, 2), (2, 3), (2, 8), (3, 4), (4, 5), (5, 2), (5, 6), (6, 7), (7, 5), (8, 1), (8, 7)
A_{323}	(1, 2), (2, 3), (3, 1), (3, 4), (3, 6), (4, 2), (4, 5), (5, 3), (6, 7), (7, 8), (8, 5)	A_{325}	(1, 2), (2, 3), (2, 7), (3, 4), (4, 2), (4, 5), (5, 6), (6, 7), (7, 4), (7, 8), (8, 2)
A_{331}	(1, 2), (2, 3), (2, 4), (2, 8), (3, 1), (4, 1), (4, 5), (5, 6), (6, 2), (6, 7), (7, 5), (8, 6)	A_{339}	(1, 2), (1, 6), (2, 3), (3, 1), (3, 4), (4, 5), (4, 8), (5, 6), (5, 7), (6, 3), (7, 4), (8, 3)
A_{341}	(1, 2), (2, 3), (2, 8), (3, 1), (3, 4), (4, 2), (4, 5), (4, 6), (5, 3), (6, 7), (7, 8), (8, 4)	A_{358}	(1, 2), (2, 3), (3, 4), (3, 7), (4, 1), (4, 5), (5, 3), (5, 6), (6, 7), (7, 5), (7, 8), (8, 3)
A_{376}	(1, 2), (2, 3), (3, 4), (3, 5), (3, 8), (4, 2), (5, 6), (5, 7), (6, 3), (7, 3), (8, 1), (8, 7)	A_{377}	(1, 2), (2, 3), (2, 6), (2, 8), (3, 1), (3, 4), (4, 5), (5, 2), (6, 5), (6, 7), (7, 2), (8, 7)
A_{384}	(1, 2), (2, 3), (2, 6), (3, 1), (3, 4), (4, 2), (4, 5), (5, 6), (5, 8), (6, 4), (6, 7), (7, 5), (8, 4)	A_{385}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 2), (4, 5), (4, 8), (5, 3), (5, 6), (6, 4), (6, 7), (7, 5), (8, 6)
A_{387}	(1, 2), (2, 3), (2, 7), (3, 1), (3, 4), (4, 2), (4, 5), (4, 6), (5, 3), (6, 7), (6, 8), (7, 4), (8, 4)	A_{391}	(1, 2), (2, 3), (2, 8), (3, 1), (3, 4), (4, 2), (4, 5), (4, 7), (5, 3), (5, 6), (6, 4), (7, 6), (8, 4)

$6(x^8 + x^7 + 2x^4 + x + 1)$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{38}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (5, 6), (6, 7), (7, 4), (7, 8)	A_{39}	(1, 2), (2, 3), (3, 4), (3, 5), (4, 2), (5, 6), (6, 7), (7, 3), (7, 8)
A_{45}	(1, 2), (2, 3), (3, 4), (4, 5), (5, 2), (5, 6), (6, 7), (7, 8), (8, 6)	A_{50}	(2, 1), (2, 3), (3, 4), (3, 6), (4, 5), (5, 2), (6, 7), (7, 3), (8, 6)
A_{56}	(2, 1), (2, 3), (3, 4), (4, 5), (5, 2), (6, 5), (6, 7), (7, 8), (8, 6)	A_{57}	(1, 2), (2, 3), (3, 4), (3, 5), (4, 2), (5, 6), (5, 8), (6, 7), (7, 3)

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{62}	(1, 2), (2, 3), (3, 4), (3, 7), (4, 5), (5, 2), (5, 6), (7, 8), (8, 3)	A_{68}	(2, 1), (2, 3), (3, 4), (3, 6), (4, 5), (5, 2), (6, 7), (7, 8), (8, 6)
A_{73}	(2, 1), (2, 3), (3, 4), (4, 5), (5, 2), (5, 6), (6, 7), (7, 5), (8, 6)	A_{75}	(2, 1), (2, 3), (3, 4), (4, 5), (5, 2), (5, 6), (6, 7), (7, 5), (8, 3)
A_{97}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 5), (5, 2), (5, 6), (6, 7), (6, 8), (7, 4)	A_{108}	(1, 2), (2, 3), (3, 1), (4, 2), (4, 5), (5, 6), (6, 7), (7, 4), (7, 8), (8, 6)
A_{114}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 5), (5, 6), (6, 7), (6, 8), (7, 5), (8, 4)	A_{115}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (5, 3), (5, 6), (6, 7), (7, 4), (8, 7)
A_{117}	(1, 2), (2, 3), (3, 4), (3, 5), (4, 2), (5, 6), (6, 7), (6, 8), (7, 3), (8, 5)	A_{138}	(1, 2), (2, 3), (2, 5), (3, 4), (4, 1), (5, 1), (6, 2), (6, 7), (7, 8), (8, 6)
A_{139}	(1, 2), (2, 3), (3, 4), (3, 5), (3, 8), (4, 2), (5, 6), (6, 7), (7, 3), (8, 7)	A_{141}	(1, 2), (2, 3), (2, 4), (4, 5), (4, 6), (5, 1), (6, 2), (6, 7), (7, 8), (8, 6)
A_{145}	(1, 2), (2, 3), (3, 4), (3, 5), (4, 2), (5, 1), (5, 6), (6, 7), (7, 5), (7, 8)	A_{147}	(1, 2), (2, 3), (3, 4), (4, 1), (4, 5), (4, 6), (5, 3), (6, 7), (7, 8), (8, 6)
A_{156}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 5), (4, 6), (5, 2), (6, 7), (6, 8), (7, 2)	A_{157}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 5), (5, 6), (6, 7), (7, 4), (7, 8), (8, 6)
A_{158}	(1, 2), (2, 3), (3, 4), (4, 2), (4, 5), (5, 6), (6, 7), (6, 8), (7, 4), (8, 5)	A_{164}	(1, 2), (2, 3), (3, 4), (3, 5), (3, 6), (4, 1), (6, 2), (6, 7), (7, 8), (8, 6)
A_{167}	(1, 2), (2, 3), (3, 4), (4, 5), (4, 6), (4, 8), (5, 2), (6, 7), (7, 4), (8, 3)	A_{180}	(1, 2), (2, 3), (3, 4), (4, 2), (4, 5), (4, 8), (5, 6), (6, 7), (7, 4), (8, 7)
A_{188}	(2, 1), (2, 3), (3, 4), (4, 2), (4, 5), (5, 6), (6, 7), (7, 4), (7, 8), (8, 6)	A_{189}	(1, 2), (2, 3), (3, 1), (4, 3), (4, 5), (5, 6), (5, 7), (6, 4), (7, 8), (8, 4)
A_{190}	(1, 2), (2, 3), (3, 1), (4, 3), (4, 5), (5, 6), (6, 7), (6, 8), (7, 4), (8, 5)	A_{191}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 5), (5, 6), (6, 2), (6, 7), (7, 5), (8, 5)
A_{194}	(2, 1), (2, 3), (2, 6), (3, 4), (4, 5), (4, 7), (5, 2), (6, 5), (7, 8), (8, 4)	A_{195}	(2, 1), (2, 3), (3, 4), (4, 2), (4, 5), (5, 6), (5, 8), (6, 7), (7, 4), (8, 4)
A_{197}	(2, 1), (2, 3), (3, 4), (3, 5), (4, 2), (5, 6), (5, 8), (6, 7), (7, 3), (8, 3)	A_{198}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (5, 6), (5, 8), (6, 7), (7, 4), (8, 4)
A_{210}	(2, 1), (2, 3), (3, 4), (3, 5), (3, 7), (4, 2), (5, 6), (6, 2), (7, 8), (8, 3)	A_{217}	(1, 2), (2, 3), (2, 8), (3, 4), (4, 5), (5, 2), (5, 6), (6, 7), (7, 5), (8, 5)
A_{228}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 1), (4, 5), (5, 2), (6, 4), (6, 7), (7, 8), (8, 6)	A_{229}	(1, 2), (2, 3), (3, 4), (3, 5), (3, 8), (4, 2), (5, 6), (6, 3), (6, 7), (7, 5), (8, 6)
A_{230}	(1, 2), (2, 3), (2, 6), (3, 4), (4, 2), (4, 5), (5, 6), (5, 7), (6, 4), (7, 8), (8, 5)	A_{233}	(2, 1), (2, 3), (3, 4), (3, 5), (4, 2), (5, 2), (5, 6), (6, 3), (6, 7), (7, 8), (8, 6)
A_{235}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (4, 6), (6, 3), (6, 7), (7, 4), (7, 8), (8, 6)	A_{245}	(1, 2), (2, 3), (3, 4), (4, 2), (4, 5), (4, 8), (5, 6), (6, 4), (6, 7), (7, 5), (8, 6)
A_{248}	(1, 2), (2, 3), (3, 1), (4, 2), (4, 5), (5, 6), (5, 7), (6, 4), (7, 4), (7, 8), (8, 5)	A_{251}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 2), (4, 5), (5, 3), (5, 6), (6, 7), (7, 5), (8, 4)
A_{253}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 1), (4, 5), (4, 6), (5, 2), (6, 7), (7, 8), (8, 6)	A_{254}	(1, 2), (2, 3), (3, 4), (3, 5), (4, 2), (5, 6), (5, 7), (6, 3), (7, 3), (7, 8), (8, 5)
A_{255}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 5), (5, 6), (5, 8), (6, 4), (6, 7), (7, 5), (8, 4)	A_{257}	(1, 2), (2, 3), (3, 4), (3, 5), (3, 7), (4, 8), (5, 6), (5, 7), (6, 4), (7, 4), (8, 7)
A_{257}	(1, 2), (2, 3), (3, 4), (3, 5), (3, 7), (4, 2), (5, 6), (6, 3), (7, 6), (7, 8), (8, 3)	A_{264}	(2, 1), (2, 3), (3, 4), (4, 2), (4, 5), (5, 6), (5, 7), (6, 4), (7, 4), (7, 8), (8, 5)
A_{284}	(2, 1), (2, 3), (2, 5), (3, 4), (4, 2), (5, 4), (5, 6), (6, 2), (6, 7), (7, 8), (8, 6)	A_{287}	(1, 2), (2, 3), (3, 1), (4, 3), (4, 5), (4, 7), (5, 6), (5, 8), (6, 4), (7, 8), (8, 4)
A_{289}	(2, 1), (2, 3), (3, 4), (3, 6), (3, 8), (4, 2), (4, 5), (5, 3), (6, 7), (7, 3), (8, 2)	A_{291}	(1, 2), (2, 3), (3, 1), (3, 4), (3, 5), (5, 2), (5, 6), (5, 7), (6, 3), (7, 8), (8, 5)
A_{292}	(1, 2), (2, 3), (3, 4), (4, 2), (4, 5), (4, 7), (5, 6), (6, 4), (7, 6), (7, 8), (8, 4)	A_{294}	(1, 2), (2, 3), (2, 5), (2, 6), (3, 1), (3, 4), (4, 2), (6, 4), (6, 7), (7, 8), (8, 6)
A_{298}	(1, 2), (2, 3), (2, 6), (3, 4), (4, 2), (4, 5), (4, 7), (5, 6), (6, 4), (7, 8), (8, 4)	A_{316}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (4, 6), (5, 3), (6, 3), (6, 7), (7, 4), (8, 4)
A_{336}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (5, 3), (5, 6), (6, 4), (6, 7), (7, 5), (7, 8), (8, 6)	A_{344}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 2), (4, 5), (5, 3), (5, 6), (5, 7), (6, 4), (7, 8), (8, 5)
A_{347}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (4, 8), (5, 3), (5, 6), (6, 4), (6, 7), (7, 8), (8, 6)	A_{359}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (4, 6), (5, 3), (6, 3), (6, 7), (7, 4), (7, 8), (8, 6)

$\mathbf{8(x^8+2x^7+2x^4+2x+1)}$	
algebra KQ/I	quiver Q
A_{91}	(1, 2), (2, 3), (2, 5), (3, 1), (4, 2), (5, 6), (6, 2), (6, 7), (7, 8), (8, 6)
A_{101}	(1, 2), (2, 3), (2, 5), (3, 1), (4, 2), (5, 6), (5, 7), (6, 2), (7, 8), (8, 5)

$\mathbf{8(x^8+x^7+x^6+2x^4+x^2+x+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{59}	(1, 2), (2, 3), (3, 4), (3, 6), (4, 5), (5, 1), (6, 7), (7, 8), (8, 6)	A_{63}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 5), (5, 6), (6, 7), (7, 8), (8, 4)
A_{64}	(1, 2), (2, 3), (3, 4), (3, 5), (4, 2), (5, 6), (6, 7), (7, 8), (8, 3)	A_{79}	(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (5, 7), (6, 2), (7, 8), (8, 5)
A_{81}	(2, 1), (2, 3), (3, 4), (4, 5), (4, 7), (5, 6), (6, 2), (7, 8), (8, 4)	A_{82}	(1, 2), (2, 3), (3, 4), (4, 2), (4, 5), (5, 6), (6, 7), (7, 8), (8, 4)
A_{130}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (5, 6), (6, 3), (6, 7), (7, 8), (8, 5)	A_{168}	(1, 2), (2, 3), (3, 4), (4, 1), (4, 5), (5, 6), (6, 3), (6, 7), (7, 8), (8, 6)
A_{236}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 5), (5, 2), (5, 6), (6, 4), (6, 7), (7, 8), (8, 5)	A_{244}	(1, 2), (2, 3), (3, 4), (4, 1), (4, 5), (5, 3), (5, 6), (5, 7), (6, 4), (7, 8), (8, 5)
A_{250}	(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (5, 6), (5, 7), (6, 3), (7, 4), (7, 8), (8, 5)	A_{259}	(1, 2), (2, 3), (2, 4), (3, 1), (4, 5), (5, 6), (5, 8), (6, 2), (6, 7), (7, 5), (8, 7)

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{269}	$(1,2),(1,6),(2,3),(3,1),(3,4),$ $(4,5),(4,7),(5,6),(6,3),(7,8),(8,4)$	A_{271}	$(1,2),(2,3),(3,4),(3,6),(4,1),$ $(4,5),(5,3),(5,7),(6,5),(7,8),(8,5)$
A_{288}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(4,7),(5,3),(5,6),(6,4),(7,8),(8,6)$	A_{301}	$(1,2),(2,3),(3,1),(3,4),(4,2),$ $(4,5),(5,6),(6,3),(6,7),(7,8),(8,6)$
A_{329}	$(1,2),(2,3),(3,1),(3,4),(3,6),(4,2),$ $(4,5),(5,3),(6,5),(6,7),(7,8),(8,6)$	A_{330}	$(1,2),(2,3),(3,1),(3,4),(3,6),(4,5),$ $(5,3),(5,7),(6,5),(7,6),(7,8),(8,5)$
A_{332}	$(1,2),(2,3),(2,4),(2,8),(3,1),(4,1),$ $(4,5),(5,2),(5,6),(6,7),(7,5),(8,5)$	A_{334}	$(1,2),(2,3),(2,5),(3,1),(3,4),(4,2),$ $(5,1),(5,6),(6,2),(6,7),(7,8),(8,6)$

D Derived equivalences for cluster-tilted algebras of type E_8

D.1 Polynomial $2(x^8 - x^6 + 2x^5 - 2x^4 + 2x^3 - x^2 + 1)$

$A_2^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_2$, $A_{19}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{28}$

A_2	$(5;4,7,8)$	$\underset{\text{der}}{\sim}$	A_{19}	$(56)(78)$	$\Rightarrow$	$A_2 \underset{\text{der}}{\sim} A_{28}$

D.2 Polynomial $2(x^8 - x^6 + x^5 + x^3 - x^2 + 1)$

$A_3^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{10}$, $A_4^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_7$, $A_5^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_5$, $A_6^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_6$, $A_{23}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{35}$, $A_{25}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{25}$, $A_{31}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{46}$

A_3	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_6	(17264358)	$\Rightarrow$	$A_{10} \underset{\text{der}}{\sim} A_6$
A_3	$(4;3,6,7)$	$\underset{\text{der}}{\sim}$	A_{23}	(56)	$\Rightarrow$	$A_{10} \underset{\text{der}}{\sim} A_{35}$
A_5^{op}	$(6;5,7)$	$\underset{\text{der}}{\sim}$	A_4	(678)	$\Rightarrow$	$A_5 \underset{\text{der}}{\sim} A_4$
A_6	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_{31}	$(17)(246)$	$\Rightarrow$	$A_6 \underset{\text{der}}{\sim} A_{46}$
A_7^{op}	$(5;3,6)$	$\underset{\text{der}}{\sim}$	A_3	$(18)(27)(3465)$	$\Rightarrow$	$A_7 \underset{\text{der}}{\sim} A_{10}$

D.3 Polynomial $3(x^8 + x^4 + 1)$

$A_8^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_9$, $A_{12}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{14}$, $A_{17}^{\mathrm{op}} = A_{17}$, $A_{26}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{34}$, $A_{30}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{33}$, $A_{43}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{60}$, $A_{44}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{66}$, $A_{47}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{67}$, $A_{53}^{\mathrm{op}} = A_{61}$, $A_{76}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{80}$, $A_{84}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{84}$, $A_{92}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{109}$, $A_{94}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{100}$, $A_{102}^{\mathrm{op}} = A_{148}$, $A_{110}^{\mathrm{op}} = A_{131}$, $A_{111}^{\mathrm{op}} = A_{171}$, $A_{123}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{123}$, $A_{132}^{\mathrm{op}} = A_{149}$, $A_{144}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{187}$, $A_{154}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{163}$, $A_{169}^{\mathrm{op}} = A_{196}$, $A_{173}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{173}$, $A_{206}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{218}$, $A_{221}^{\mathrm{op}} = A_{222}$, $A_{232}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{242}$, $A_{247}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{277}$, $A_{272}^{\mathrm{op}} = A_{275}$, $A_{305}^{\mathrm{op}} = A_{305}$

A_8	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{100}	$(45)(678)$	$\Rightarrow$	$A_9 \underset{\text{der}}{\sim} A_{94}$
A_9	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{109}	(576)	$\Rightarrow$	$A_8 \underset{\text{der}}{\sim} A_{92}$
A_{12}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{154}	(465)	$\Rightarrow$	$A_{14} \underset{\text{der}}{\sim} A_{163}$
A_{14}	$(6;5,7)$	$\underset{\text{der}}{\sim}$	A_{196}	$(18)(275)(46)$	$\Rightarrow$	$A_{12} \underset{\text{der}}{\sim} A_{169}$
A_{17}	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{218}	(185236)	$\Rightarrow$	$A_{17} \underset{\text{der}}{\sim} A_{206}$
$A_{44}(*)$	$(4;3,7,8)$	$\underset{\text{der}}{\sim}$	A_{92}	(567)	$\Rightarrow$	$A_{66} \underset{\text{der}}{\sim} A_{109}$
A_{92}	$(7;6)$	$\underset{\text{der}}{\sim}$	A_{43}	(578)	$\Rightarrow$	$A_{109} \underset{\text{der}}{\sim} A_{60}$
A_{100}	$(8;4)$	$\underset{\text{der}}{\sim}$	A_{26}	(78)	$\Rightarrow$	$A_{94} \underset{\text{der}}{\sim} A_{34}$
$A_{102}(*)$	$(4;3,6,7)$	$\underset{\text{der}}{\sim}$	A_{43}	(567)	$\Rightarrow$	$A_{148} \underset{\text{der}}{\sim} A_{60}$
A_{102}	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_{132}	(35674)	$\Rightarrow$	$A_{148} \underset{\text{der}}{\sim} A_{149}$
A_{109}	$(6;5)$	$\underset{\text{der}}{\sim}$	A_{30}	(56)	$\Rightarrow$	$A_{92} \underset{\text{der}}{\sim} A_{33}$
A_{123}	$(4;3,6,8)$	$\underset{\text{der}}{\sim}$	A_{163}	(4587)	$\Rightarrow$	$A_{123} \underset{\text{der}}{\sim} A_{154}$
$A_{131}(*)$	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_{53}	(4756)	$\Rightarrow$	$A_{110} \underset{\text{der}}{\sim} A_{61}$
$A_{132}(*)$	$(3;2,4,7)$	$\underset{\text{der}}{\sim}$	A_{247}	(1)	$\Rightarrow$	$A_{149} \underset{\text{der}}{\sim} A_{277}$
A_{154}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{47}	(45)	$\Rightarrow$	$A_{163} \underset{\text{der}}{\sim} A_{67}$
$A_{154}(*)$	$(3;2,5,6)$	$\underset{\text{der}}{\sim}$	A_{84}	$(34)(576)$	$\Rightarrow$	$A_{163} \underset{\text{der}}{\sim} A_{84}$
$A_{173}(*)$	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_{218}	(34657)	$\Rightarrow$	$A_{173} \underset{\text{der}}{\sim} A_{206}$
$A_{187}(*)$	$(5;4,6,8)$	$\underset{\text{der}}{\sim}$	A_{154}	(485)	$\Rightarrow$	$A_{144} \underset{\text{der}}{\sim} A_{163}$
A_{196}	$(6;3)$	$\underset{\text{der}}{\sim}$	A_{61}	$(18)(267)$	$\Rightarrow$	$A_{169} \underset{\text{der}}{\sim} A_{53}$
$A_{196}(*)$	$(2;1,5,7)$	$\underset{\text{der}}{\sim}$	A_{80}	$(178)(243)(56)$	$\Rightarrow$	$A_{169} \underset{\text{der}}{\sim} A_{76}$
$A_{218}(*)$	$(4;3,5,7)$	$\underset{\text{der}}{\sim}$	A_{196}	(37654)	$\Rightarrow$	$A_{206} \underset{\text{der}}{\sim} A_{169}$

A_{232}	$(5;4,7)$	$\underset{\text{der}}{\sim}$	A_{44}	(48675)	$\Rightarrow$	$A_{242} \underset{\text{der}}{\sim} A_{66}$
A_{247}	$(6;5,8)$	$\underset{\text{der}}{\sim}$	A_{53}	$(47)(56)$	$\Rightarrow$	$A_{277} \underset{\text{der}}{\sim} A_{61}$
A_{272}	$(4;3,6)$	$\underset{\text{der}}{\sim}$	A_{84}	(387564)	$\Rightarrow$	$A_{275} \underset{\text{der}}{\sim} A_{84}$
$A_{275}(*)$	$(7;3,6,8)$	$\underset{\text{der}}{\sim}$	A_{111}	$(183546)(27)$	$\Rightarrow$	$A_{272} \underset{\text{der}}{\sim} A_{171}$
A_{305}	$(5;4,7,8)$	$\underset{\text{der}}{\sim}$	A_{222}	(1876423)	$\Rightarrow$	$A_{305} \underset{\text{der}}{\sim} A_{221}$
A_{305}	$(6;2,5)$	$\underset{\text{der}}{\sim}$	A_{80}	$(18)(24635)$	$\Rightarrow$	$A_{305} \underset{\text{der}}{\sim} A_{76}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

D.4 Polynomial $4(x^8 + x^7 - x^6 + x^5 + x^3 - x^2 + x + 1)$

$A_{20}^{\text{op}} \underset{\text{s/s}}{\sim} A_{41}$, $A_{21}^{\text{op}} = A_{49}$, $A_{22}^{\text{op}} \underset{\text{s/s}}{\sim} A_{52}$, $A_{27}^{\text{op}} \underset{\text{s/s}}{\sim} A_{27}$, $A_{29}^{\text{op}} = A_{36}$, $A_{37}^{\text{op}} \underset{\text{s/s}}{\sim} A_{37}$, $A_{89}^{\text{op}} = A_{122}$, $A_{90}^{\text{op}} \underset{\text{s/s}}{\sim} A_{142}$, $A_{98}^{\text{op}} \underset{\text{s/s}}{\sim} A_{106}$, $A_{105}^{\text{op}} \underset{\text{s/s}}{\sim} A_{124}$

A_{20}	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_{27}	$(18765)(23)$	$\Rightarrow$	$A_{41} \underset{\text{der}}{\sim} A_{27}$
A_{21}	$(2;1,4,5)$	$\underset{\text{der}}{\sim}$	A_{90}	$(18)(2647)(35)$	$\Rightarrow$	$A_{49} \underset{\text{der}}{\sim} A_{142}$
A_{22}	$(2;1,4,5)$	$\underset{\text{der}}{\sim}$	A_{89}	(34)	$\Rightarrow$	$A_{52} \underset{\text{der}}{\sim} A_{122}$
A_{36}	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_{20}	(45)	$\Rightarrow$	$A_{29} \underset{\text{der}}{\sim} A_{41}$
A_{37}^{op}	$(6;5,7)$	$\underset{\text{der}}{\sim}$	A_{21}	$(23)(678)$	$\Rightarrow$	$A_{37} \underset{\text{der}}{\sim} A_{49}$
A_{37}^{op}	$(2;3,4,5)$	$\underset{\text{der}}{\sim}$	A_{124}	$(143)(78)$	$\Rightarrow$	$A_{37} \underset{\text{der}}{\sim} A_{105}$
A_{49}	$(5;2,7)$	$\underset{\text{der}}{\sim}$	A_{27}	$(178)(24536)$	$\Rightarrow$	$A_{21} \underset{\text{der}}{\sim} A_{27}$
A_{52}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{36}	(34)	$\Rightarrow$	$A_{22} \underset{\text{der}}{\sim} A_{29}$
A_{122}	$(6;5,8)$	$\underset{\text{der}}{\sim}$	A_{106}	(67)	$\Rightarrow$	$A_{89} \underset{\text{der}}{\sim} A_{98}$

D.5 Polynomial $4(x^8 + x^7 - x^6 + 2x^4 - x^2 + x + 1)$

$A_{24}^{\text{op}} \underset{\text{s/s}}{\sim} A_{32}$, $A_{93}^{\text{op}} = A_{121}$, $A_{107}^{\text{op}} \underset{\text{s/s}}{\sim} A_{153}$, $A_{113}^{\text{op}} \underset{\text{s/s}}{\sim} A_{152}$, $A_{120}^{\text{op}} = A_{146}$, $A_{137}^{\text{op}} = A_{155}$

A_{32}	$(5;3,8)$	$\underset{\text{der}}{\sim}$	A_{24}	(5687)		
A_{32}	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_{113}	$(18267)(345)$	$\Rightarrow$	$A_{24} \underset{\text{der}}{\sim} A_{152}$
A_{93}	$(6;4,8)$	$\underset{\text{der}}{\sim}$	A_{107}	(67)	$\Rightarrow$	$A_{121} \underset{\text{der}}{\sim} A_{153}$
A_{113}	$(3;2,5,6)$	$\underset{\text{der}}{\sim}$	A_{107}	$(13478)(26)$	$\Rightarrow$	$A_{152} \underset{\text{der}}{\sim} A_{153}$
A_{120}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{153}	$(18)(26547)$	$\Rightarrow$	$A_{146} \underset{\text{der}}{\sim} A_{107}$
A_{155}	$(6;3,8)$	$\underset{\text{der}}{\sim}$	A_{120}	$(18)(27456)$	$\Rightarrow$	$A_{137} \underset{\text{der}}{\sim} A_{146}$

D.6 Polynomial $4(x^8 + x^7 - 2x^6 + 2x^5 + 2x^3 - 2x^2 + x + 1)$

$A_{95}^{\text{op}} = A_{119}$, $A_{96}^{\text{op}} \underset{\text{s/s}}{\sim} A_{116}$

A_{96}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{116}	(134)		
A_{119}	$(6;5,8)$	$\underset{\text{der}}{\sim}$	A_{96}	$(18)(2746)(35)$	$\Rightarrow$	$A_{95} \underset{\text{der}}{\sim} A_{116}$

D.7 Polynomial $4(x^8 + x^6 - x^5 + 2x^4 - x^3 + x^2 + 1)$

$A_{11}^{\text{op}} \underset{\text{s/s}}{\sim} A_{11}$, $A_{13}^{\text{op}} \underset{\text{s/s}}{\sim} A_{16}$, $A_{40}^{\text{op}} \underset{\text{s/s}}{\sim} A_{40}$, $A_{42}^{\text{op}} \underset{\text{s/s}}{\sim} A_{55}$, $A_{54}^{\text{op}} = A_{54}$, $A_{58}^{\text{op}} \underset{\text{s/s}}{\sim} A_{72}$, $A_{85}^{\text{op}} = A_{87}$, $A_{99}^{\text{op}} \underset{\text{s/s}}{\sim} A_{128}$, $A_{103}^{\text{op}} \underset{\text{s/s}}{\sim} A_{162}$, $A_{104}^{\text{op}} \underset{\text{s/s}}{\sim} A_{136}$, $A_{112}^{\text{op}} \underset{\text{s/s}}{\sim} A_{129}$, $A_{126}^{\text{op}} \underset{\text{s/s}}{\sim} A_{143}$, $A_{127}^{\text{op}} \underset{\text{s/s}}{\sim} A_{185}$, $A_{133}^{\text{op}} = A_{150}$, $A_{134}^{\text{op}} \underset{\text{s/s}}{\sim} A_{134}$, $A_{135}^{\text{op}} \underset{\text{s/s}}{\sim} A_{182}$, $A_{151}^{\text{op}} = A_{172}$, $A_{170}^{\text{op}} = A_{175}$, $A_{176}^{\text{op}} = A_{177}$, $A_{186}^{\text{op}} \underset{\text{s/s}}{\sim} A_{193}$, $A_{192}^{\text{op}} \underset{\text{s/s}}{\sim} A_{207}$, $A_{208}^{\text{op}} \underset{\text{s/s}}{\sim} A_{208}$, $A_{224}^{\text{op}} \underset{\text{s/s}}{\sim} A_{231}$, $A_{225}^{\text{op}} = A_{239}$, $A_{226}^{\text{op}} = A_{237}$, $A_{227}^{\text{op}} \underset{\text{s/s}}{\sim} A_{258}$, $A_{238}^{\text{op}} = A_{240}$, $A_{243}^{\text{op}} \underset{\text{s/s}}{\sim} A_{286}$, $A_{273}^{\text{op}} = A_{276}$, $A_{282}^{\text{op}} = A_{304}$, $A_{311}^{\text{op}} = A_{324}$, $A_{333}^{\text{op}} = A_{363}$, $A_{337}^{\text{op}} \underset{\text{s/s}}{\sim} A_{338}$, $A_{342}^{\text{op}} \underset{\text{s/s}}{\sim} A_{343}$, $A_{352}^{\text{op}} = A_{370}$, $A_{361}^{\text{op}} = A_{366}$, $A_{388}^{\text{op}} = A_{388}$

A_{11}	$(4;3,8)$	$\underset{\text{der}}{\sim}$	A_{162}	$(45)(678)$	$\Rightarrow$	$A_{11} \underset{\text{der}}{\sim} A_{103}$
A_{13}	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{192}	$(34)(567)$	$\Rightarrow$	$A_{16} \underset{\text{der}}{\sim} A_{207}$
A_{13}	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{112}	(576)	$\Rightarrow$	$A_{16} \underset{\text{der}}{\sim} A_{129}$
A_{40}^{op}	$(2;1,3)$	$\underset{\text{der}}{\sim}$	A_{258}	$(18)(27)(35)$	$\Rightarrow$	$A_{40} \underset{\text{der}}{\sim} A_{227}$

A_{42}	$(3;2,6)$	$\underset{\text{der}}{\sim}$	A_{243}	$(1827)(3645)$	$\Rightarrow$	$A_{55} \underset{\text{der}}{\sim} A_{286}$
A_{42}	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{224}	$(56)(78)$	$\Rightarrow$	$A_{55} \underset{\text{der}}{\sim} A_{231}$
A_{42}^{op}	$(6;3,7)$	$\underset{\text{der}}{\sim}$	A_{239}	(48765)	$\Rightarrow$	$A_{42} \underset{\text{der}}{\sim} A_{225}$
A_{54}	$(6;4,8)$	$\underset{\text{der}}{\sim}$	A_{276}	(587)	$\Rightarrow$	$A_{54} \underset{\text{der}}{\sim} A_{273}$
A_{54}	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{226}	$(354)(67)$	$\Rightarrow$	$A_{54} \underset{\text{der}}{\sim} A_{237}$
$A_{58}(*)$	$(3;2,7,8)$	$\underset{\text{der}}{\sim}$	A_{104}	(4567)	$\Rightarrow$	$A_{72} \underset{\text{der}}{\sim} A_{136}$
A_{58}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{143}	(458)	$\Rightarrow$	$A_{72} \underset{\text{der}}{\sim} A_{126}$
$A_{85}(*)$	$(5;4,7,8)$	$\underset{\text{der}}{\sim}$	A_{176}	$(56)(78)$	$\Rightarrow$	$A_{87} \underset{\text{der}}{\sim} A_{177}$
A_{87}	$(3;2)$	$\underset{\text{der}}{\sim}$	A_{208}	$(18)(246375)$	$\Rightarrow$	$A_{85} \underset{\text{der}}{\sim} A_{208}$
A_{99}^{op}	$(5;6,7,8)$	$\underset{\text{der}}{\sim}$	A_{237}	$(17)(2635)$	$\Rightarrow$	$A_{99} \underset{\text{der}}{\sim} A_{226}$
A_{104}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{129}	(468)	$\Rightarrow$	$A_{136} \underset{\text{der}}{\sim} A_{112}$
A_{112}	$(6;5)$	$\underset{\text{der}}{\sim}$	A_{225}	(687)	$\Rightarrow$	$A_{129} \underset{\text{der}}{\sim} A_{239}$
A_{112}	$(3;2,6)$	$\underset{\text{der}}{\sim}$	A_{333}	$(34)(56)$	$\Rightarrow$	$A_{129} \underset{\text{der}}{\sim} A_{363}$
A_{127}	$(6;5,8)$	$\underset{\text{der}}{\sim}$	A_{126}	(15362478)	$\Rightarrow$	$A_{185} \underset{\text{der}}{\sim} A_{143}$
A_{127}^{op}	$(2;1,3,8)$	$\underset{\text{der}}{\sim}$	A_{238}	$(18)(46)$	$\Rightarrow$	$A_{127} \underset{\text{der}}{\sim} A_{240}$
A_{134}	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{342}	(5876)	$\Rightarrow$	$A_{134} \underset{\text{der}}{\sim} A_{343}$
A_{135}	$(5;4,7)$	$\underset{\text{der}}{\sim}$	A_{103}	(56)	$\Rightarrow$	$A_{182} \underset{\text{der}}{\sim} A_{162}$
A_{135}	$(3,2,6)$	$\underset{\text{der}}{\sim}$	A_{343}	$(1845)(2736)$	$\Rightarrow$	$A_{182} \underset{\text{der}}{\sim} A_{342}$
A_{162}	$(7;4)$	$\underset{\text{der}}{\sim}$	A_{231}	(78)	$\Rightarrow$	$A_{103} \underset{\text{der}}{\sim} A_{224}$
A_{170}	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{361}	$(34)(5867)$	$\Rightarrow$	$A_{175} \underset{\text{der}}{\sim} A_{366}$
A_{170}	$(6;4,8)$	$\underset{\text{der}}{\sim}$	A_{150}	$(18)(274356)$	$\Rightarrow$	$A_{175} \underset{\text{der}}{\sim} A_{133}$
$A_{172}(*)$	$(2;1,4,7)$	$\underset{\text{der}}{\sim}$	A_{282}	(134)	$\Rightarrow$	$A_{151} \underset{\text{der}}{\sim} A_{304}$
A_{176}	$(4;3,6)$	$\underset{\text{der}}{\sim}$	A_{133}	(45)	$\Rightarrow$	$A_{177} \underset{\text{der}}{\sim} A_{150}$
$A_{186}(*)$	$(3;2,6,8)$	$\underset{\text{der}}{\sim}$	A_{363}	$(1625)(34)$	$\Rightarrow$	$A_{193} \underset{\text{der}}{\sim} A_{333}$
A_{208}	$(4;3,7,8)$	$\underset{\text{der}}{\sim}$	A_{342}	(4576)	$\Rightarrow$	$A_{208} \underset{\text{der}}{\sim} A_{343}$
A_{226}	$(7;3,8)$	$\underset{\text{der}}{\sim}$	A_{240}	(1)	$\Rightarrow$	$A_{237} \underset{\text{der}}{\sim} A_{238}$
A_{258}	$(3;2,6)$	$\underset{\text{der}}{\sim}$	A_{231}	(456)	$\Rightarrow$	$A_{227} \underset{\text{der}}{\sim} A_{224}$
A_{282}	$(7;2,6)$	$\underset{\text{der}}{\sim}$	A_{311}	$(38)(4657)$	$\Rightarrow$	$A_{304} \underset{\text{der}}{\sim} A_{324}$
$A_{324}(*)$	$(2;1,4,6)$	$\underset{\text{der}}{\sim}$	A_{366}	$(134)(687)$	$\Rightarrow$	$A_{311} \underset{\text{der}}{\sim} A_{361}$
A_{337}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{352}	(23)	$\Rightarrow$	$A_{338} \underset{\text{der}}{\sim} A_{370}$
A_{352}	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{333}	(345)	$\Rightarrow$	$A_{370} \underset{\text{der}}{\sim} A_{363}$
A_{388}	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{324}	(3465)	$\Rightarrow$	$A_{388} \underset{\text{der}}{\sim} A_{311}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

D.8 Polynomial $4(x^8 + x^6 - 2x^5 + 4x^4 - 2x^3 + x^2 + 1)$

$A_{48}^{\text{op}} \underset{\text{s/s}}{\sim} A_{70}$, $A_{118}^{\text{op}} \underset{\text{s/s}}{\sim} A_{160}$, $A_{161}^{\text{op}} \underset{\text{s/s}}{\sim} A_{161}$, $A_{278}^{\text{op}} = A_{302}$

A_{48}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{118}	(56)	$\Rightarrow$	$A_{70} \underset{\text{der}}{\sim} A_{160}$
$A_{70}(*)$	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{302}	(576)	$\Rightarrow$	$A_{48} \underset{\text{der}}{\sim} A_{278}$
A_{161}	$(2;1,6)$	$\underset{\text{der}}{\sim}$	A_{160}	(3456)	$\Rightarrow$	$A_{161} \underset{\text{der}}{\sim} A_{118}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

D.9 Polynomial $5(x^8 + x^6 + x^4 + x^2 + 1)$

$A_{18}^{\text{op}} = A_{18}$, $A_{51}^{\text{op}} \underset{\text{s/s}}{\sim} A_{69}$, $A_{65}^{\text{op}} = A_{71}$, $A_{74}^{\text{op}} \underset{\text{s/s}}{\sim} A_{78}$, $A_{77}^{\text{op}} = A_{86}$, $A_{83}^{\text{op}} \underset{\text{s/s}}{\sim} A_{83}$, $A_{125}^{\text{op}} = A_{159}$, $A_{140}^{\text{op}} = A_{174}$, $A_{165}^{\text{op}} \underset{\text{s/s}}{\sim} A_{166}$, $A_{178}^{\text{op}} = A_{204}$, $A_{181}^{\text{op}} = A_{202}$, $A_{183}^{\text{op}} \underset{\text{s/s}}{\sim} A_{200}$, $A_{199}^{\text{op}} \underset{\text{s/s}}{\sim} A_{203}$, $A_{201}^{\text{op}} \underset{\text{s/s}}{\sim} A_{201}$, $A_{212}^{\text{op}} \underset{\text{s/s}}{\sim} A_{220}$, $A_{213}^{\text{op}} \underset{\text{s/s}}{\sim} A_{219}$, $A_{214}^{\text{op}} \underset{\text{s/s}}{\sim} A_{216}$, $A_{223}^{\text{op}} = A_{223}$, $A_{234}^{\text{op}} \underset{\text{s/s}}{\sim} A_{234}$, $A_{241}^{\text{op}} = A_{274}$, $A_{246}^{\text{op}} = A_{265}$, $A_{249}^{\text{op}} = A_{260}$, $A_{252}^{\text{op}} \underset{\text{s/s}}{\sim} A_{266}$, $A_{261}^{\text{op}} \underset{\text{s/s}}{\sim} A_{285}$, $A_{262}^{\text{op}} \underset{\text{s/s}}{\sim} A_{295}$, $A_{263}^{\text{op}} \underset{\text{s/s}}{\sim} A_{293}$, $A_{267}^{\text{op}} = A_{281}$, $A_{279}^{\text{op}} = A_{303}$, $A_{283}^{\text{op}} \underset{\text{s/s}}{\sim} A_{296}$, $A_{297}^{\text{op}} = A_{310}$, $A_{306}^{\text{op}} = A_{307}$, $A_{312}^{\text{op}} \underset{\text{s/s}}{\sim} A_{314}$, $A_{315}^{\text{op}} = A_{318}$, $A_{321}^{\text{op}} = A_{327}$, $A_{322}^{\text{op}} = A_{328}$, $A_{326}^{\text{op}} = A_{326}$, $A_{335}^{\text{op}} \underset{\text{s/s}}{\sim} A_{356}$, $A_{340}^{\text{op}} = A_{348}$, $A_{345}^{\text{op}} = A_{355}$, $A_{346}^{\text{op}} = A_{364}$, $A_{349}^{\text{op}} = A_{360}$, $A_{350}^{\text{op}} = A_{357}$, $A_{351}^{\text{op}} \underset{\text{s/s}}{\sim} A_{353}$, $A_{354}^{\text{op}} = A_{371}$, $A_{362}^{\text{op}} = A_{365}$, $A_{367}^{\text{op}} = A_{375}$, $A_{368}^{\text{op}} = A_{372}$, $A_{369}^{\text{op}} = A_{378}$, $A_{373}^{\text{op}} = A_{373}$, $A_{374}^{\text{op}} = A_{381}$, $A_{379}^{\text{op}} = A_{380}$,

$A_{382}^{\mathrm{op}} = A_{383}$, $A_{386}^{\mathrm{op}} = A_{390}$, $A_{389}^{\mathrm{op}} = A_{389}$

A_{18}	$(2;1,7)$	$\widetilde{\text{der}}$	A_{216}	$(18)(274635)$	$\Rightarrow$	$A_{18} \widetilde{\text{der}} A_{240}$
$A_{51}(*)$	$(2;1,7)$	$\widetilde{\text{der}}$	A_{281}	$(12)(678)$	$\Rightarrow$	$A_{69} \widetilde{\text{der}} A_{267}$
A_{51}	$(6;5)$	$\widetilde{\text{der}}$	A_{125}	(687)	$\Rightarrow$	$A_{69} \widetilde{\text{der}} A_{159}$
$A_{74}(*)$	$(3;2,8)$	$\widetilde{\text{der}}$	A_{318}	$(1236)(78)$	$\Rightarrow$	$A_{78} \widetilde{\text{der}} A_{315}$
A_{74}	$(4;3)$	$\widetilde{\text{der}}$	A_{201}	$(1654)(2783)$	$\Rightarrow$	$A_{78} \widetilde{\text{der}} A_{201}$
A_{77}	$(6;4)$	$\widetilde{\text{der}}$	A_{181}	(67)	$\Rightarrow$	$A_{86} \widetilde{\text{der}} A_{202}$
A_{77}	$(5;4)$	$\widetilde{\text{der}}$	A_{140}	(38765)	$\Rightarrow$	$A_{86} \widetilde{\text{der}} A_{174}$
$A_{78}(*)$	$(2;1,7,8)$	$\widetilde{\text{der}}$	A_{140}	(1834567)	$\Rightarrow$	$A_{74} \widetilde{\text{der}} A_{174}$
A_{78}	$(4;3)$	$\widetilde{\text{der}}$	A_{166}	(16548)	$\Rightarrow$	$A_{74} \widetilde{\text{der}} A_{165}$
A_{83}	$(2;1,6)$	$\widetilde{\text{der}}$	A_{322}	$(1827)(3654)$	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{328}$
A_{83}	$(6;5)$	$\widetilde{\text{der}}$	A_{212}	$(386)(475)$	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{220}$
A_{83}	$(7;5)$	$\widetilde{\text{der}}$	A_{216}	(78)	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{214}$
A_{83}	$(4;3,8)$	$\widetilde{\text{der}}$	A_{293}	(48765)	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{263}$
A_{125}	$(7;5)$	$\widetilde{\text{der}}$	A_{71}	(567)	$\Rightarrow$	$A_{159} \widetilde{\text{der}} A_{65}$
$A_{125}(*)$	$(2;1,6)$	$\widetilde{\text{der}}$	A_{350}	$(17)(264)(35)$	$\Rightarrow$	$A_{159} \widetilde{\text{der}} A_{357}$
A_{140}	$(8;2,7)$	$\widetilde{\text{der}}$	A_{183}	(5768)	$\Rightarrow$	$A_{174} \widetilde{\text{der}} A_{200}$
$A_{165}(*)$	$(2;1,4)$	$\widetilde{\text{der}}$	A_{371}	$(174385)(26)$	$\Rightarrow$	$A_{166} \widetilde{\text{der}} A_{354}$
A_{165}	$(7;2)$	$\widetilde{\text{der}}$	A_{295}	$(17)(38)(456)$	$\Rightarrow$	$A_{166} \widetilde{\text{der}} A_{262}$
A_{166}	$(7;3)$	$\widetilde{\text{der}}$	A_{51}	(34567)	$\Rightarrow$	$A_{165} \widetilde{\text{der}} A_{69}$
$A_{166}(*)$	$(2;1,6,7)$	$\widetilde{\text{der}}$	A_{246}	$(187)(3456)$	$\Rightarrow$	$A_{165} \widetilde{\text{der}} A_{265}$
A_{178}	$(2;1,6)$	$\widetilde{\text{der}}$	A_{199}	(374856)	$\Rightarrow$	$A_{204} \widetilde{\text{der}} A_{203}$
A_{178}	$(5;4,8)$	$\widetilde{\text{der}}$	A_{365}	$(182637)(45)$	$\Rightarrow$	$A_{204} \widetilde{\text{der}} A_{362}$
A_{178}	$(7;6)$	$\widetilde{\text{der}}$	A_{281}	$(16358)(247)$	$\Rightarrow$	$A_{204} \widetilde{\text{der}} A_{367}$
A_{213}	$(3;2,7)$	$\widetilde{\text{der}}$	A_{379}	(346758)	$\Rightarrow$	$A_{219} \widetilde{\text{der}} A_{380}$
A_{213}	$(8;3)$	$\widetilde{\text{der}}$	A_{77}	(1)	$\Rightarrow$	$A_{219} \widetilde{\text{der}} A_{86}$
A_{216}	$(6;5)$	$\widetilde{\text{der}}$	A_{321}	(37546)	$\Rightarrow$	$A_{214} \widetilde{\text{der}} A_{327}$
A_{216}	$(2;1,6)$	$\widetilde{\text{der}}$	A_{382}	$(23)(456)$	$\Rightarrow$	$A_{214} \widetilde{\text{der}} A_{383}$
A_{223}	$(3;2,6)$	$\widetilde{\text{der}}$	A_{381}	$(34)(5876)$	$\Rightarrow$	$A_{223} \widetilde{\text{der}} A_{374}$
A_{223}	$(1;4)$	$\widetilde{\text{der}}$	A_{322}	$(1827)(35)(46)$	$\Rightarrow$	$A_{223} \widetilde{\text{der}} A_{328}$
$A_{234}(*)$	$(5;4,6,8)$	$\widetilde{\text{der}}$	A_{293}	(4685)	$\Rightarrow$	$A_{234} \widetilde{\text{der}} A_{263}$
A_{241}	$(8;5)$	$\widetilde{\text{der}}$	A_{178}	(132)	$\Rightarrow$	$A_{274} \widetilde{\text{der}} A_{204}$
A_{249}	$(4;3,6)$	$\widetilde{\text{der}}$	A_{285}	(13247568)	$\Rightarrow$	$A_{260} \widetilde{\text{der}} A_{261}$
$A_{260}(*)$	$(2;1,4,7)$	$\widetilde{\text{der}}$	A_{345}	$(134)(687)$	$\Rightarrow$	$A_{249} \widetilde{\text{der}} A_{355}$
A_{260}	$(4;3)$	$\widetilde{\text{der}}$	A_{125}	$(34)(67)$	$\Rightarrow$	$A_{249} \widetilde{\text{der}} A_{159}$
A_{279}	$(3;2,5)$	$\widetilde{\text{der}}$	A_{241}	(18274536)	$\Rightarrow$	$A_{303} \widetilde{\text{der}} A_{274}$
A_{281}	$(4;3,6)$	$\widetilde{\text{der}}$	A_{252}	(17542861)	$\Rightarrow$	$A_{267} \widetilde{\text{der}} A_{266}$
A_{283}	$(3;2,5,7)$	$\widetilde{\text{der}}$	A_{261}	$(18)(267)(34)$	$\Rightarrow$	$A_{296} \widetilde{\text{der}} A_{285}$
A_{293}	$(5;4)$	$\widetilde{\text{der}}$	A_{351}	(38765)	$\Rightarrow$	$A_{263} \widetilde{\text{der}} A_{353}$
A_{293}	$(6;4)$	$\widetilde{\text{der}}$	A_{200}	(68)	$\Rightarrow$	$A_{263} \widetilde{\text{der}} A_{183}$
A_{293}	$(2;1,5)$	$\widetilde{\text{der}}$	A_{389}	(18435726)	$\Rightarrow$	$A_{263} \widetilde{\text{der}} A_{389}$
A_{297}	$(1;4)$	$\widetilde{\text{der}}$	A_{362}	$(18)(2736)$	$\Rightarrow$	$A_{310} \widetilde{\text{der}} A_{365}$
A_{306}	$(7;4,8)$	$\widetilde{\text{der}}$	A_{65}	(15437268)	$\Rightarrow$	$A_{307} \widetilde{\text{der}} A_{71}$
A_{312}	$(4;3)$	$\widetilde{\text{der}}$	A_{183}	(34)	$\Rightarrow$	$A_{314} \widetilde{\text{der}} A_{200}$
A_{326}	$(8;3,7)$	$\widetilde{\text{der}}$	A_{328}	$(172846)(35)$	$\Rightarrow$	$A_{326} \widetilde{\text{der}} A_{322}$
$A_{340}(*)$	$(6;5,7)$	$\widetilde{\text{der}}$	A_{360}	$(148)(25)$	$\Rightarrow$	$A_{348} \widetilde{\text{der}} A_{349}$
A_{346}	$(7;2,6)$	$\widetilde{\text{der}}$	A_{362}	(57)	$\Rightarrow$	$A_{364} \widetilde{\text{der}} A_{365}$
A_{350}	$(2;1,5)$	$\widetilde{\text{der}}$	A_{335}	$(18536)(27)$	$\Rightarrow$	$A_{357} \widetilde{\text{der}} A_{356}$
A_{360}	$(1;3)$	$\widetilde{\text{der}}$	A_{306}	$(12)(48)(576)$	$\Rightarrow$	$A_{349} \widetilde{\text{der}} A_{307}$
A_{367}	$(4;3,5)$	$\widetilde{\text{der}}$	A_{178}	$(18)(25)(3746)$	$\Rightarrow$	$A_{375} \widetilde{\text{der}} A_{204}$
A_{368}	$(8;3)$	$\widetilde{\text{der}}$	A_{283}	$(1827)(46)$	$\Rightarrow$	$A_{372} \widetilde{\text{der}} A_{296}$
A_{369}	$(5;4)$	$\widetilde{\text{der}}$	A_{386}	$(132)(46758)$	$\Rightarrow$	$A_{378} \widetilde{\text{der}} A_{390}$
A_{369}	$(7;6,8)$	$\widetilde{\text{der}}$	A_{373}	(17428536)	$\Rightarrow$	$A_{378} \widetilde{\text{der}} A_{373}$

A_{369}	$(4;3,7)$	$\underset{\mathrm{der}}{\sim}$	A_{354}	$(16)(287)(35)$	$\Rightarrow$	$A_{378} \underset{\mathrm{der}}{\sim} A_{371}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

D.10 Polynomial $6(x^8+x^6+x^5+x^3+x^2+1)$

$A_{15}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{15}$, $A_{88}^{\mathrm{op}} = A_{88}$, $A_{179}^{\mathrm{op}} = A_{184}$, $A_{205}^{\mathrm{op}} = A_{211}$, $A_{209}^{\mathrm{op}} = A_{215}$, $A_{268}^{\mathrm{op}} = A_{299}$, $A_{270}^{\mathrm{op}} = A_{280}$, $A_{290}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{319}$, $A_{300}^{\mathrm{op}} = A_{308}$, $A_{309}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{317}$, $A_{313}^{\mathrm{op}} = A_{323}$, $A_{320}^{\mathrm{op}} = A_{320}$, $A_{325}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{325}$, $A_{331}^{\mathrm{op}} = A_{339}$, $A_{341}^{\mathrm{op}} = A_{358}$, $A_{376}^{\mathrm{op}} = A_{377}$, $A_{384}^{\mathrm{op}} = A_{385}$, $A_{387}^{\mathrm{op}} = A_{391}$

A_{15}^{op}	$(2;1,3)$	$\underset{\mathrm{der}}{\sim}$	A_{184}	$(28)(37)(46)$	$\Rightarrow$	$A_{15} \underset{\mathrm{der}}{\sim} A_{179}$
A_{88}	$(2;1,8)$	$\underset{\mathrm{der}}{\sim}$	A_{323}	$(124)(5678)$	$\Rightarrow$	$A_{88} \underset{\mathrm{der}}{\sim} A_{313}$
A_{184}	$(3;2)$	$\underset{\mathrm{der}}{\sim}$	A_{280}	(1238)	$\Rightarrow$	$A_{179} \underset{\mathrm{der}}{\sim} A_{270}$
A_{184}	$(7;6,8)$	$\underset{\mathrm{der}}{\sim}$	A_{215}	$(123658)(47)$	$\Rightarrow$	$A_{179} \underset{\mathrm{der}}{\sim} A_{209}$
A_{205}	$(4;3,7)$	$\underset{\mathrm{der}}{\sim}$	A_{341}	(16542738)	$\Rightarrow$	$A_{211} \underset{\mathrm{der}}{\sim} A_{358}$
A_{211}	$(3;2,8)$	$\underset{\mathrm{der}}{\sim}$	A_{377}	$(142536)(78)$	$\Rightarrow$	$A_{205} \underset{\mathrm{der}}{\sim} A_{376}$
A_{280}	$(7;1,6)$	$\underset{\mathrm{der}}{\sim}$	A_{308}	(4657)	$\Rightarrow$	$A_{270} \underset{\mathrm{der}}{\sim} A_{300}$
A_{290}	$(3;2,5)$	$\underset{\mathrm{der}}{\sim}$	A_{309}	$(18)(267)(34)$	$\Rightarrow$	$A_{319} \underset{\mathrm{der}}{\sim} A_{317}$
A_{299}	$(5;4,8)$	$\underset{\mathrm{der}}{\sim}$	A_{391}	$(186)(243)$	$\Rightarrow$	$A_{268} \underset{\mathrm{der}}{\sim} A_{387}$
A_{308}	$(5;4,7)$	$\underset{\mathrm{der}}{\sim}$	A_{313}	$(1234)(67)$	$\Rightarrow$	$A_{300} \underset{\mathrm{der}}{\sim} A_{323}$
A_{317}^{op}	$(5;2,6)$	$\underset{\mathrm{der}}{\sim}$	A_{325}	(386547)	$\Rightarrow$	$A_{309} \underset{\mathrm{der}}{\sim} A_{325}$
A_{320}	$(7;6,8)$	$\underset{\mathrm{der}}{\sim}$	A_{319}	(14725836)	$\Rightarrow$	$A_{320} \underset{\mathrm{der}}{\sim} A_{290}$
A_{323}	$(6;3)$	$\underset{\mathrm{der}}{\sim}$	A_{377}	$(1846)(273)$	$\Rightarrow$	$A_{313} \underset{\mathrm{der}}{\sim} A_{376}$
A_{325}	$(2;1,4,8)$	$\underset{\mathrm{der}}{\sim}$	A_{323}	$(1468)(23)(57)$	$\Rightarrow$	$A_{325} \underset{\mathrm{der}}{\sim} A_{313}$
A_{339}	$(6;1,5)$	$\underset{\mathrm{der}}{\sim}$	A_{358}	(476)	$\Rightarrow$	$A_{331} \underset{\mathrm{der}}{\sim} A_{341}$
A_{384}	$(4;3,6,8)$	$\underset{\mathrm{der}}{\sim}$	A_{300}	(34)	$\Rightarrow$	$A_{385} \underset{\mathrm{der}}{\sim} A_{308}$
A_{387}	$(4;3,7,8)$	$\underset{\mathrm{der}}{\sim}$	A_{377}	(1425786)	$\Rightarrow$	$A_{391} \underset{\mathrm{der}}{\sim} A_{376}$

D.11 Polynomial $6(x^8+x^7+2x^4+x+1)$

$A_{38}^{\mathrm{op}} = A_{45}$, $A_{39}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{73}$, $A_{50}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{57}$, $A_{56}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{68}$, $A_{62}^{\mathrm{op}} = A_{75}$, $A_{97}^{\mathrm{op}} = A_{115}$, $A_{108}^{\mathrm{op}} = A_{114}$, $A_{117}^{\mathrm{op}} = A_{188}$, $A_{138}^{\mathrm{op}} = A_{189}$, $A_{139}^{\mathrm{op}} = A_{195}$, $A_{141}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{164}$, $A_{145}^{\mathrm{op}} = A_{158}$, $A_{147}^{\mathrm{op}} = A_{198}$, $A_{156}^{\mathrm{op}} = A_{167}$, $A_{157}^{\mathrm{op}} = A_{190}$, $A_{180}^{\mathrm{op}} = A_{197}$, $A_{191}^{\mathrm{op}} = A_{194}$, $A_{210}^{\mathrm{op}} = A_{217}$, $A_{228}^{\mathrm{op}} = A_{248}$, $A_{229}^{\mathrm{op}} = A_{264}$, $A_{230}^{\mathrm{op}} = A_{233}$, $A_{235}^{\mathrm{op}} = A_{251}$, $A_{245}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{254}$, $A_{253}^{\mathrm{op}} = A_{255}$, $A_{256}^{\mathrm{op}} = A_{287}$, $A_{257}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{292}$, $A_{284}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{294}$, $A_{289}^{\mathrm{op}} = A_{298}$, $A_{291}^{\mathrm{op}} = A_{316}$, $A_{336}^{\mathrm{op}} = A_{347}$, $A_{344}^{\mathrm{op}} = A_{359}$

A_{38}	$(4;3,7)$	$\underset{\mathrm{der}}{\sim}$	A_{233}	$(1728)(3645)$	$\Rightarrow$	$A_{45} \underset{\mathrm{der}}{\sim} A_{230}$
$A_{39}(*)$	$(7;6,8)$	$\underset{\mathrm{der}}{\sim}$	A_{257}	(1)	$\Rightarrow$	$A_{73} \underset{\mathrm{der}}{\sim} A_{292}$
A_{45}	$(6;5,8)$	$\underset{\mathrm{der}}{\sim}$	A_{57}	$(18)(2536)(47)$	$\Rightarrow$	$A_{38} \underset{\mathrm{der}}{\sim} A_{50}$
A_{62}	$(2;1,5)$	$\underset{\mathrm{der}}{\sim}$	A_{289}	(1524876)	$\Rightarrow$	$A_{75} \underset{\mathrm{der}}{\sim} A_{298}$
A_{68}	$(6;3,8)$	$\underset{\mathrm{der}}{\sim}$	A_{39}	$(18)(27456)$	$\Rightarrow$	$A_{56} \underset{\mathrm{der}}{\sim} A_{73}$
$A_{68}(*)$	$(2;1,5)$	$\underset{\mathrm{der}}{\sim}$	A_{287}	(16347258)	$\Rightarrow$	$A_{56} \underset{\mathrm{der}}{\sim} A_{256}$
A_{75}	$(3;2,8)$	$\underset{\mathrm{der}}{\sim}$	A_{235}	(1537246)	$\Rightarrow$	$A_{62} \underset{\mathrm{der}}{\sim} A_{251}$
A_{108}	$(2;1,4)$	$\underset{\mathrm{der}}{\sim}$	A_{145}	$(1845)(2736)$	$\Rightarrow$	$A_{114} \underset{\mathrm{der}}{\sim} A_{158}$
A_{114}	$(8;6)$	$\underset{\mathrm{der}}{\sim}$	A_{228}	$(185)(26)(37)$	$\Rightarrow$	$A_{108} \underset{\mathrm{der}}{\sim} A_{248}$
A_{115}	$(6;5)$	$\underset{\mathrm{der}}{\sim}$	A_{230}	$(1728)(3546)$	$\Rightarrow$	$A_{97} \underset{\mathrm{der}}{\sim} A_{233}$
A_{117}	$(2;1,4)$	$\underset{\mathrm{der}}{\sim}$	A_{158}	(134)	$\Rightarrow$	$A_{188} \underset{\mathrm{der}}{\sim} A_{145}$
A_{139}	$(5;3)$	$\underset{\mathrm{der}}{\sim}$	A_{257}	$(58)(67)$	$\Rightarrow$	$A_{195} \underset{\mathrm{der}}{\sim} A_{292}$
A_{139}	$(2;1,4)$	$\underset{\mathrm{der}}{\sim}$	A_{180}	(134)	$\Rightarrow$	$A_{195} \underset{\mathrm{der}}{\sim} A_{197}$
A_{141}	$(2;1,3,6)$	$\underset{\mathrm{der}}{\sim}$	A_{253}	$(24)(35)$	$\Rightarrow$	$A_{164} \underset{\mathrm{der}}{\sim} A_{255}$
A_{141}	$(5;4)$	$\underset{\mathrm{der}}{\sim}$	A_{235}	$(17)(246358)$	$\Rightarrow$	$A_{164} \underset{\mathrm{der}}{\sim} A_{251}$
A_{147}	$(1;4)$	$\underset{\mathrm{der}}{\sim}$	A_{287}	(1638257)	$\Rightarrow$	$A_{198} \underset{\mathrm{der}}{\sim} A_{256}$
$A_{156}(*)$	$(6;4,8)$	$\underset{\mathrm{der}}{\sim}$	A_{359}	$(123)(67)$	$\Rightarrow$	$A_{167} \underset{\mathrm{der}}{\sim} A_{344}$
A_{157}	$(4;2,7)$	$\underset{\mathrm{der}}{\sim}$	A_{347}	$(123)(45)(687)$	$\Rightarrow$	$A_{190} \underset{\mathrm{der}}{\sim} A_{336}$

A_{189}	$(3;2,4)$	$\underset{\text{der}}{\sim}$	A_{197}	$(1432)(687)$	$\Rightarrow$	$A_{138} \underset{\text{der}}{\sim} A_{180}$
A_{191}	$(5;4,7,8)$	$\underset{\text{der}}{\sim}$	A_{230}	$(1837)(25)(46)$	$\Rightarrow$	$A_{194} \underset{\text{der}}{\sim} A_{233}$
A_{228}	$(1;3,4)$	$\underset{\text{der}}{\sim}$	A_{38}	$(17)(2638)$	$\Rightarrow$	$A_{248} \underset{\text{der}}{\sim} A_{45}$
A_{229}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{245}	(134)	$\Rightarrow$	$A_{264} \underset{\text{der}}{\sim} A_{254}$
A_{229}	$(7;6)$	$\underset{\text{der}}{\sim}$	A_{139}	(67)	$\Rightarrow$	$A_{264} \underset{\text{der}}{\sim} A_{195}$
A_{257}	$(6;5,7)$	$\underset{\text{der}}{\sim}$	A_{57}	(5876)	$\Rightarrow$	$A_{292} \underset{\text{der}}{\sim} A_{50}$
A_{284}	$(4;3,5)$	$\underset{\text{der}}{\sim}$	A_{75}	(387654)	$\Rightarrow$	$A_{294} \underset{\text{der}}{\sim} A_{62}$
A_{291}	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{62}	$(35)(46)$	$\Rightarrow$	$A_{316} \underset{\text{der}}{\sim} A_{75}$
A_{291}	$(1;3)$	$\underset{\text{der}}{\sim}$	A_{210}	$(164)(253)$	$\Rightarrow$	$A_{316} \underset{\text{der}}{\sim} A_{217}$
$A_{291}(*)$	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_{284}	(145632)	$\Rightarrow$	$A_{316} \underset{\text{der}}{\sim} A_{294}$
A_{336}	$(5;4,7)$	$\underset{\text{der}}{\sim}$	A_{141}	$(17436)(285)$	$\Rightarrow$	$A_{347} \underset{\text{der}}{\sim} A_{164}$
A_{344}	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_{210}	(146532)	$\Rightarrow$	$A_{359} \underset{\text{der}}{\sim} A_{217}$
A_{359}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{189}	(5687)	$\Rightarrow$	$A_{344} \underset{\text{der}}{\sim} A_{138}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

D.12 Polynomial $8(x^8+2x^7+2x^4+2x+1)$

$A_{91}^{\text{op}} \underset{\text{s/s}}{\sim} A_{101}$

A_{91}	$(6;5,8)$	$\underset{\text{der}}{\sim}$	A_{101}	(5786)

D.13 Polynomial $8(x^8+x^7+x^6+2x^4+x^2+x+1)$

$A_{59}^{\text{op}} = A_{63}$, $A_{64}^{\text{op}} = A_{82}$, $A_{79}^{\text{op}} = A_{81}$, $A_{130}^{\text{op}} = A_{168}$, $A_{236}^{\text{op}} = A_{288}$, $A_{244}^{\text{op}} = A_{271}$, $A_{250}^{\text{op}} = A_{259}$, $A_{269}^{\text{op}} = A_{301}$, $A_{329}^{\text{op}} = A_{334}$, $A_{330}^{\text{op}} = A_{332}$

A_{59}	$(6;3,8)$	$\underset{\text{der}}{\sim}$	A_{64}	(1745628)	$\Rightarrow$	$A_{63} \underset{\text{der}}{\sim} A_{82}$
A_{63}	$(4;2,8)$	$\underset{\text{der}}{\sim}$	A_{288}	$(123)(45)(678)$	$\Rightarrow$	$A_{59} \underset{\text{der}}{\sim} A_{236}$
A_{64}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{82}	(134)		
A_{79}	$(2;1,6)$	$\underset{\text{der}}{\sim}$	A_{250}	$(1827)(35)(46)$	$\Rightarrow$	$A_{81} \underset{\text{der}}{\sim} A_{259}$
A_{244}	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_{259}	$(18)(273645)$	$\Rightarrow$	$A_{271} \underset{\text{der}}{\sim} A_{250}$
A_{250}	$(6;5)$	$\underset{\text{der}}{\sim}$	A_{330}	(46)	$\Rightarrow$	$A_{259} \underset{\text{der}}{\sim} A_{332}$
A_{269}	$(6;1,5)$	$\underset{\text{der}}{\sim}$	A_{236}	(1735428)	$\Rightarrow$	$A_{301} \underset{\text{der}}{\sim} A_{288}$
A_{288}	$(7;4)$	$\underset{\text{der}}{\sim}$	A_{334}	(1742836)	$\Rightarrow$	$A_{236} \underset{\text{der}}{\sim} A_{329}$
A_{330}	$(6;3,7)$	$\underset{\text{der}}{\sim}$	A_{130}	(1)	$\Rightarrow$	$A_{332} \underset{\text{der}}{\sim} A_{168}$
A_{334}	$(1;3,5)$	$\underset{\text{der}}{\sim}$	A_{168}	(1423)	$\Rightarrow$	$A_{329} \underset{\text{der}}{\sim} A_{130}$

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